L'anonimo, Soddy, il teorema

Come disegnare un grappolo di sfere, facendo in modo che ciascuna baci le vicine

Felice Ragazzo

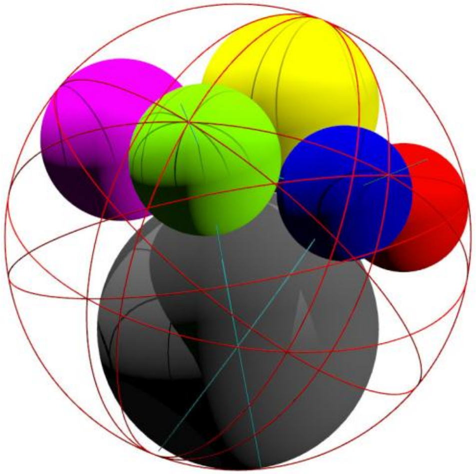
designer

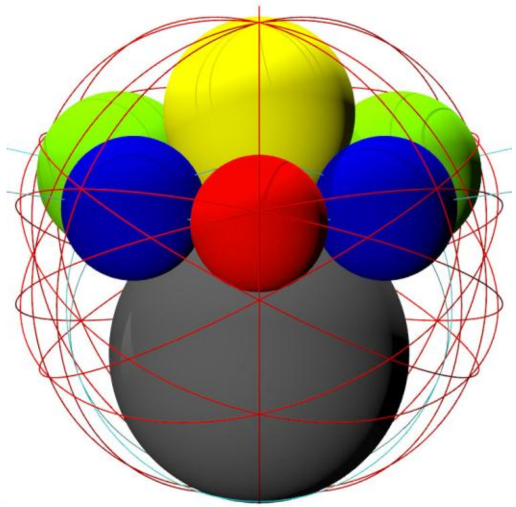
Il **teorema di Soddy**

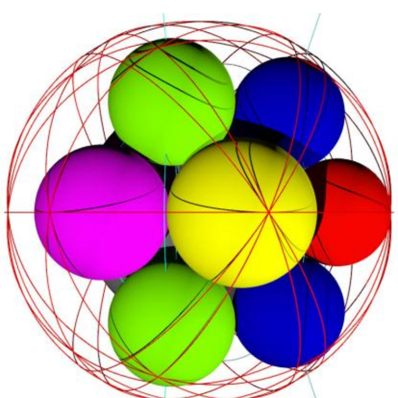
detto anche **teorema del collare**.

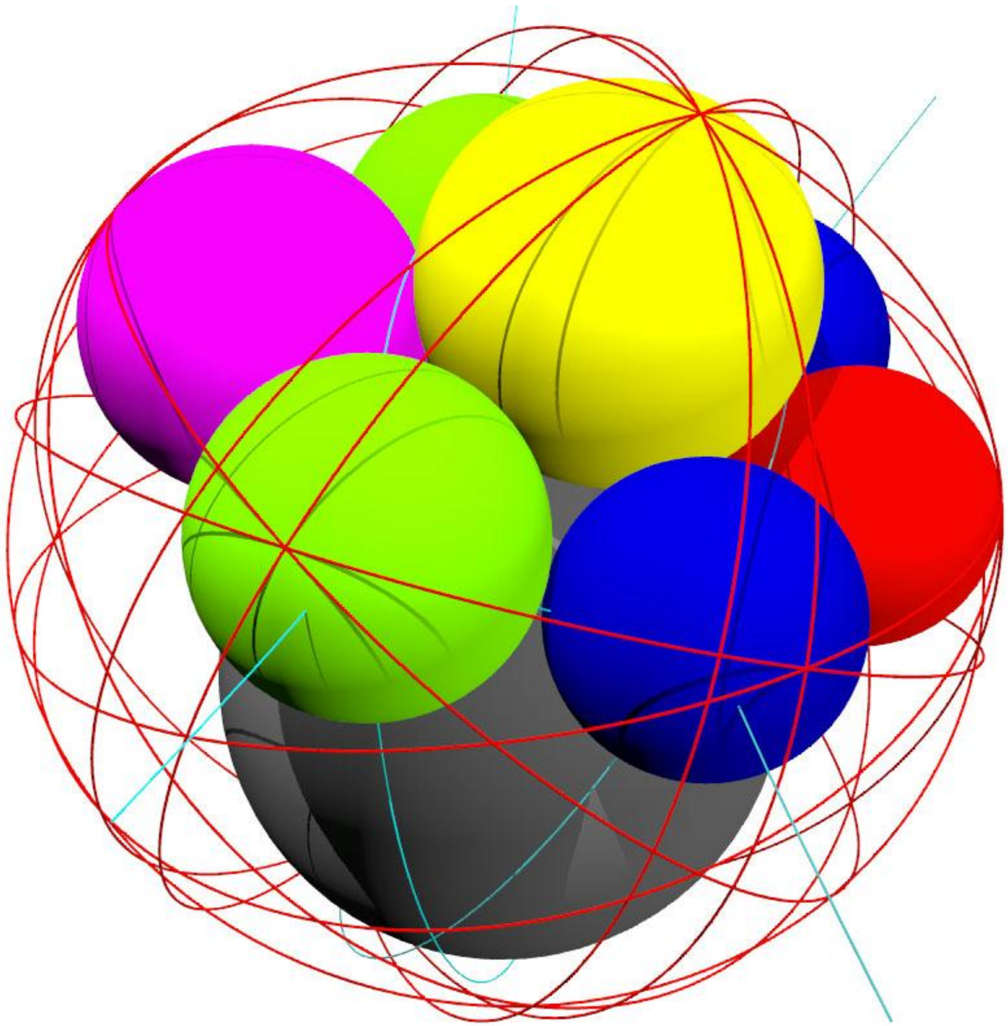
Consiste in **nove sfere** tutte tangenti tra loro

Le **nove sfere**









Tavoletta lignea votiva presso un tempio sito nella Prefettura di Kanagawa (Giappone)

1830 circa

L’anonimo, Soddy, il teorema

Tony ROTHMAN est professeur de mathématiques à l’Université Harvard. Hidetoshi FUKAGAWA est professeur de mathématiques dans la préfecture d’Aichi.

Annick HORIUCHI, *Les mathématiques japonaises à l’époque d’Edo (1600-1868), une étude des travaux de Seki Takakazu et de Takebe Katahiro*, Librairie philosophique J. Vrin, Paris, 1994.

Yoshio MIKAMI, *The Development of Mathematics in China and Japan*, deuxième édition, Chelsea Publishing Company, New York, 1974.

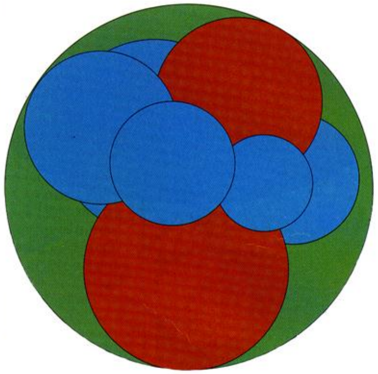
H. FUKAGAWA et D. PEDOE, *Japanese Temple Geometry Problems*, Charles Babbage Research Foundation, Winnipeg, Canada, 1989.

David E. SMITH et Yoshio MIKAMI, *A History of Japanese Mathematics*, Open Court Publishing Company, Chicago, 1914 (également disponible sur microfilm).

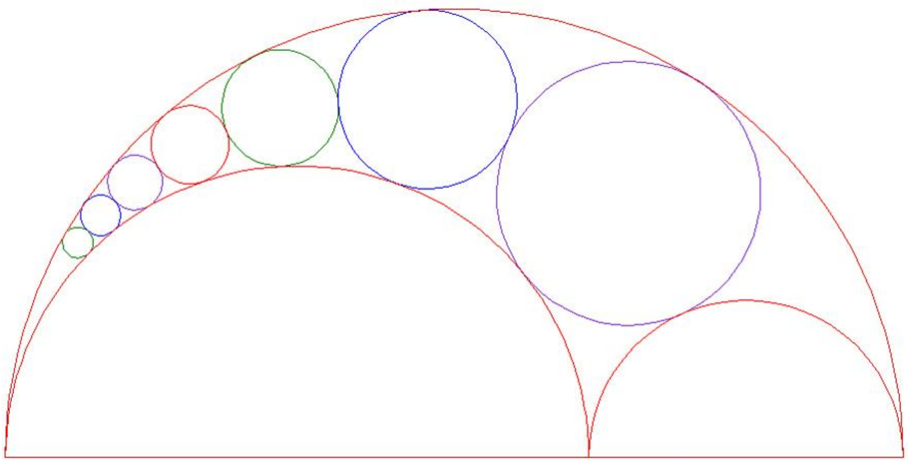
H. FUKAGAWA et D. SOKOLOWSKY, *Traditional Japanese Mathematics Problems from the 18th and 19th Centuries*, Science Culture Technology Publishing, Singapour, à paraître.

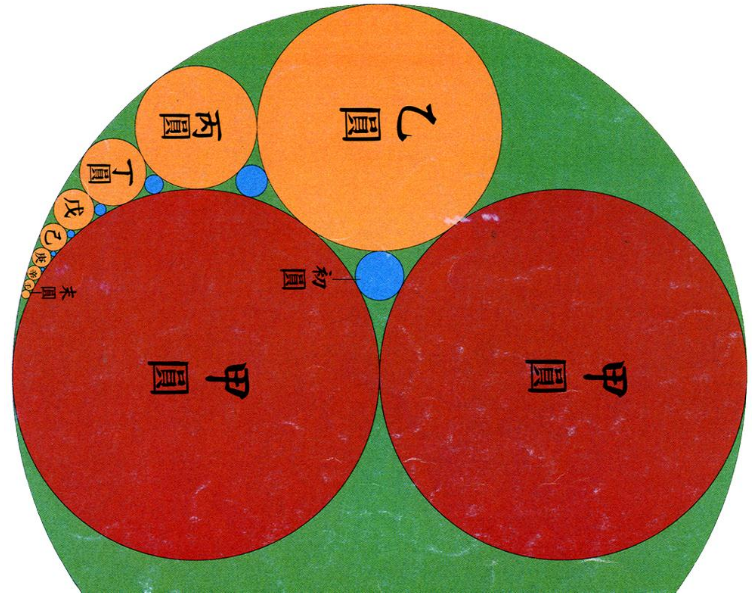
Ce problème était mentionné sur une tablette de 1822, retrouvée dans la préfecture de Kanagawa. Il anticipe de plus d’un siècle un théorème de Frederick Soddy, le chimiste britannique qui, avec Ernest Rutherford, découvrit la transmutation des éléments chimiques. Deux sphères rouges tangentes entre elles sont intérieurement tangentes à la grande sphère verte. Un «collier» de sphères bleues plus petites et de tailles différentes entoure le point de tangence des deux sphères rouges. Les sphères bleues du collier sont deux à deux tangentes, tangentes aux sphères rouges et tangentes à la sphère verte. Quel est le nombre des sphères bleues ? Quelle relation unit les rayons des sphères bleues ?

Réponse : Six sphères. Le théorème des six sphères de Soddy stipule que le nombre de sphères est égale à six. Curieusement, les rayons $t_1, t_2, ..., t_6$ des diverses sphères bleues qui composent le collier sont liés par les relations : $1/t_1 + 1/t_4 = 1/t_2 + 1/t_5 = 1/t_3 + 1/t_6$.



L’anonimo, Soddy, il teorema





Conosciuto questo problema, mi sono proposto di risolverlo

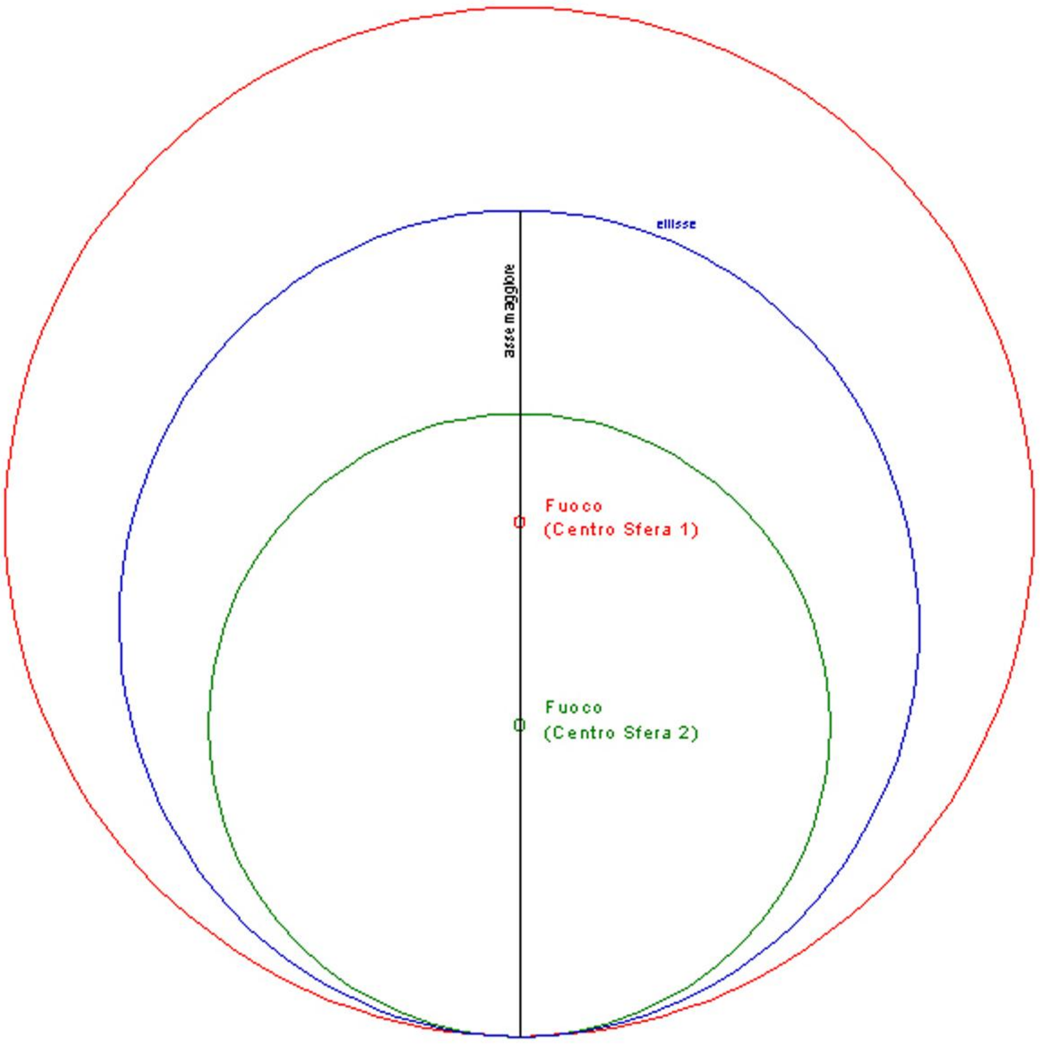
La tecnica che ho adottato si basa sulle curve policentriche

L’anonimo, Soddy, il teorema

Il luogo di equidistanza tra due circonferenze tangenti interne è un’**ellisse** dalle seguenti proprietà:

• i **fuochi** coincidono con i **centri** delle circonferenze stesse

• l’asse maggiore misura la distanza compresa tra il **punto di tangenza** e quello **intermedio** tra i quadranti opposti



L’anonimo, Soddy, il teorema

In ogni ellisse **a+b = k** (asse maggiore)

r1 = a+r

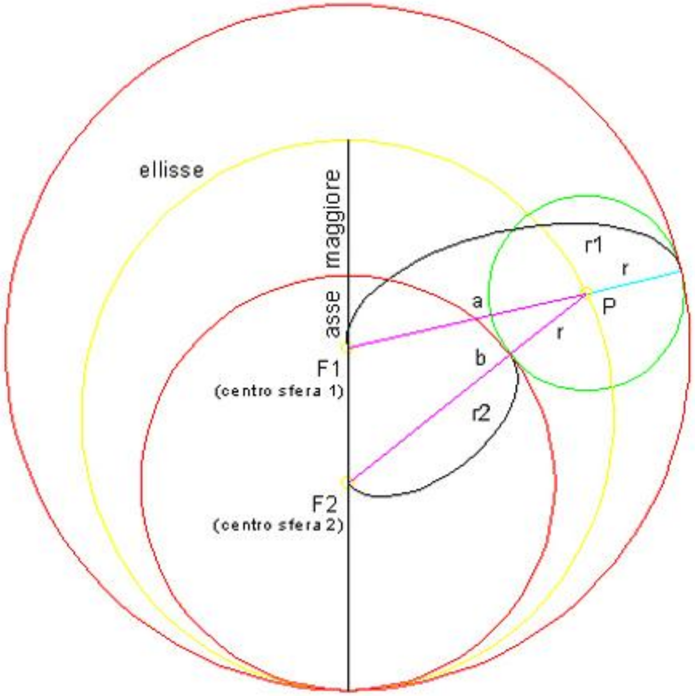
r2 = b-r

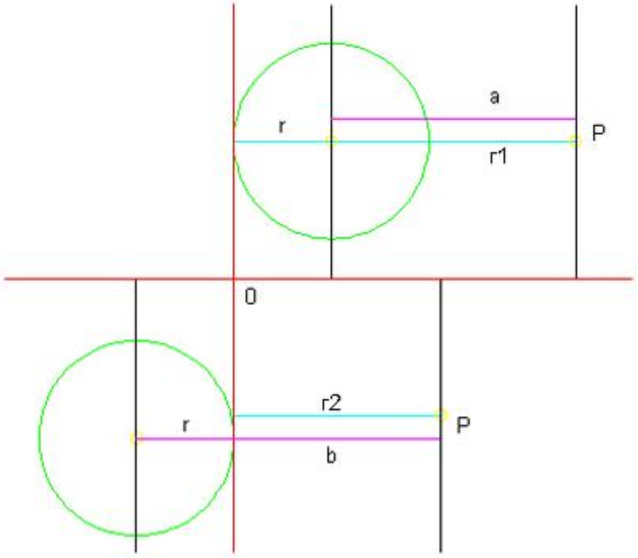
r1-a = r

r2-b = -r (val. ass.)

LdE equidistante

dai cerchi r1 e r2



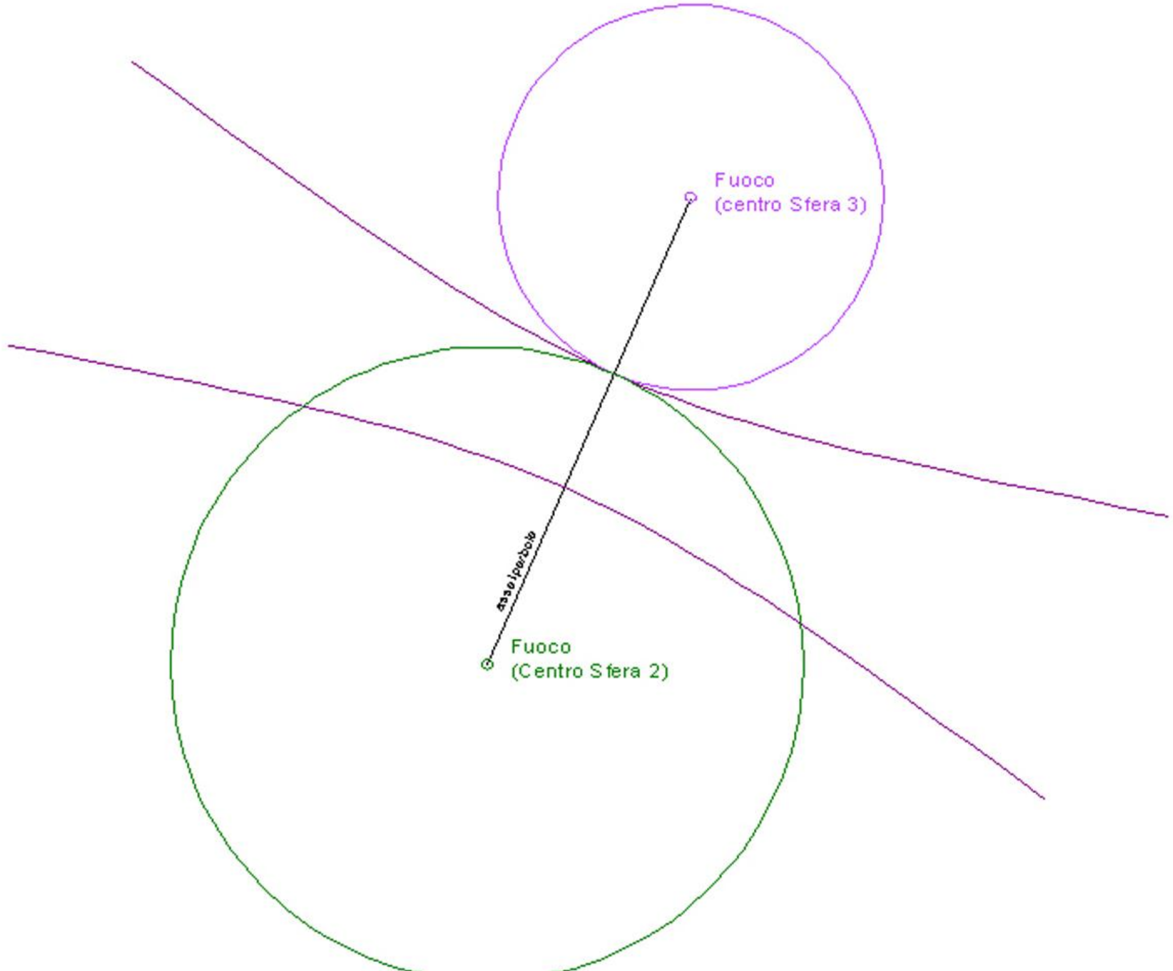


L’anonimo, Soddy, il teorema

Il luogo di equidistanza tra due circonferenze tangenti esterne è un’**iperbole** dalle seguenti proprietà:

• i **fuochi** coincidono con i **centri** delle circonferenze stesse

• un **ramo** passa per il **punto di tangenza**



L’anonimo, Soddy, il teorema

In ogni iperbole **a+b = k**

r1 = a-r

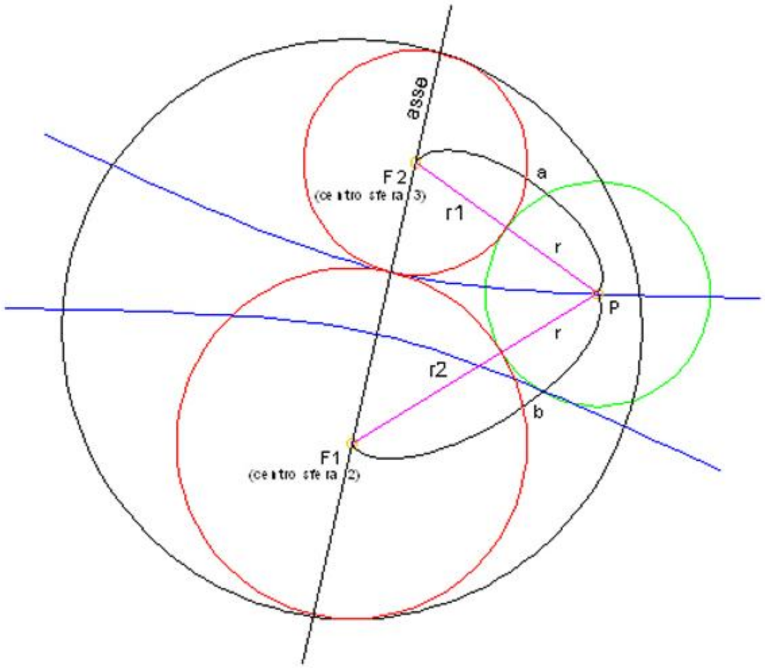
r2 = b-r

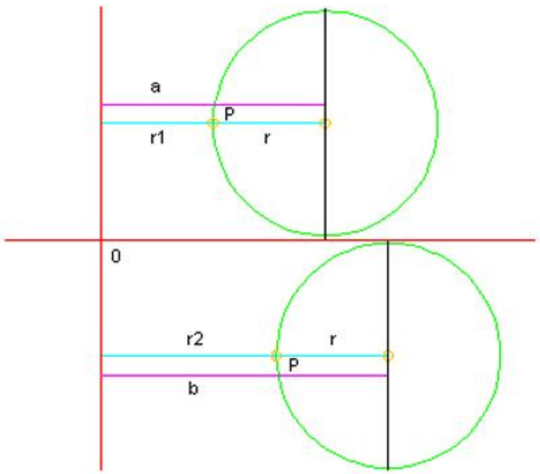
r1-a = -r

r2-b = -r

LdE equidistante

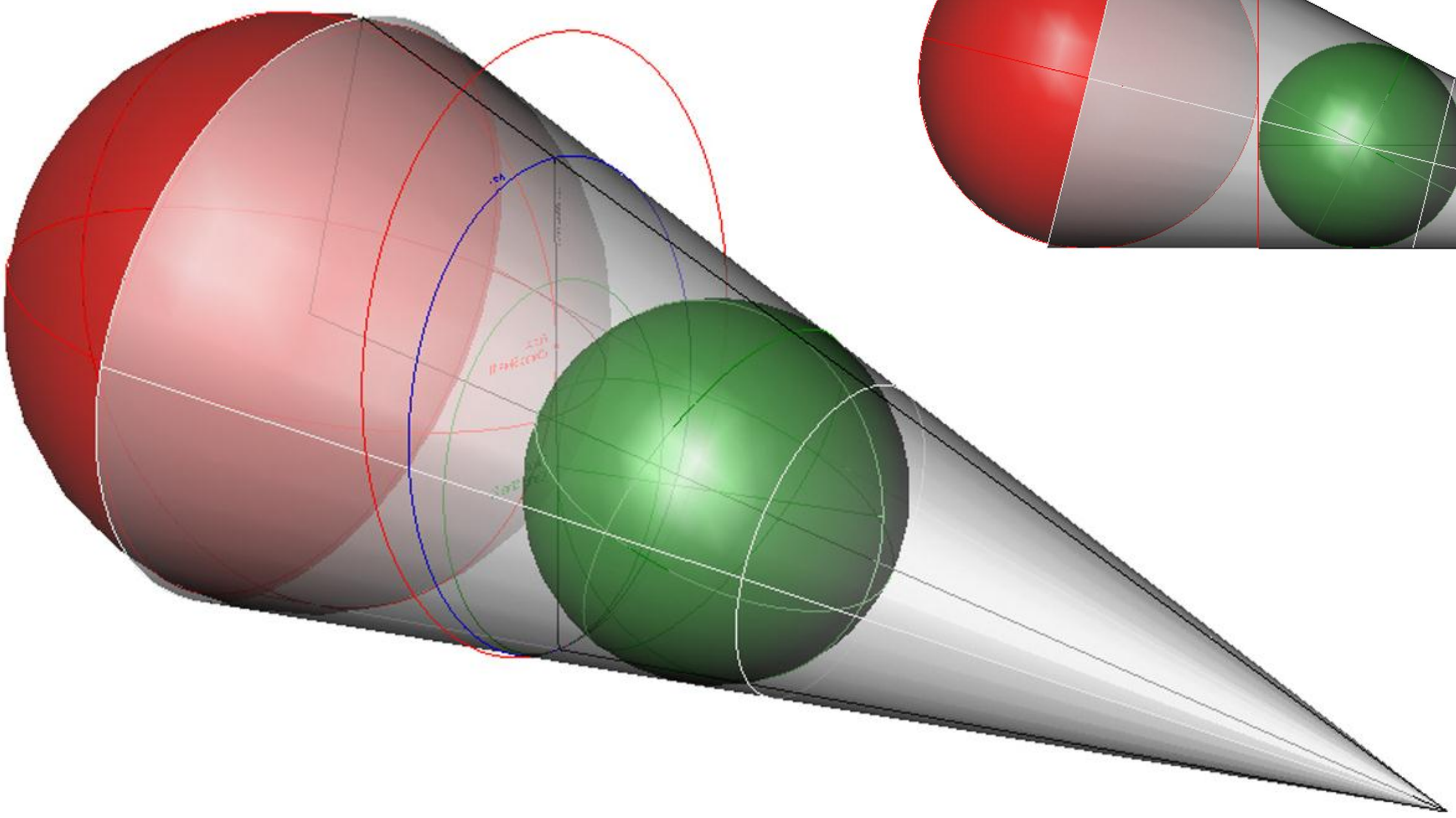
dai cerchi r1 e r2

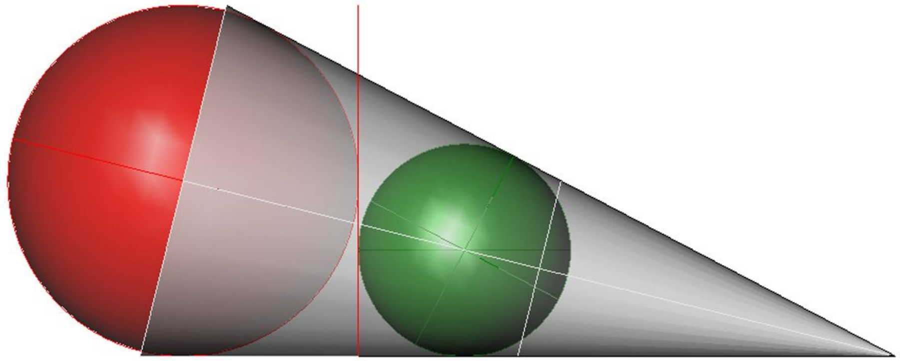




Le **sfere** di **Dandelin**

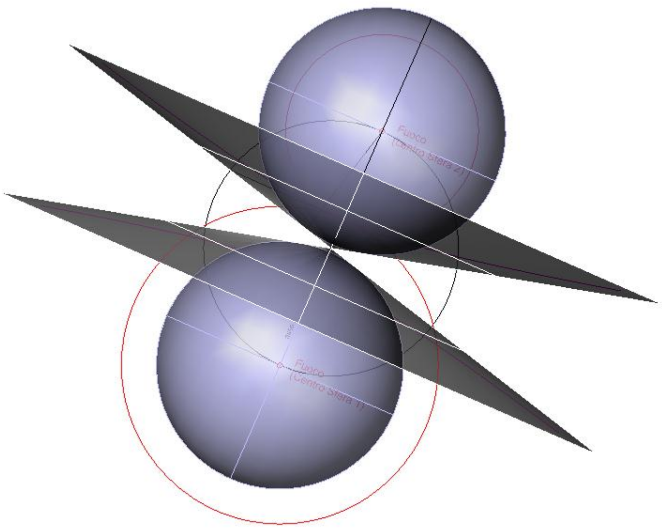
- caso dell'**ellisse**

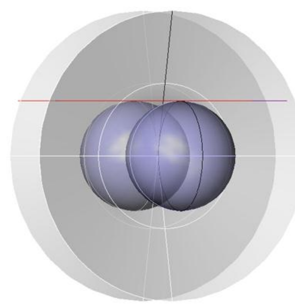


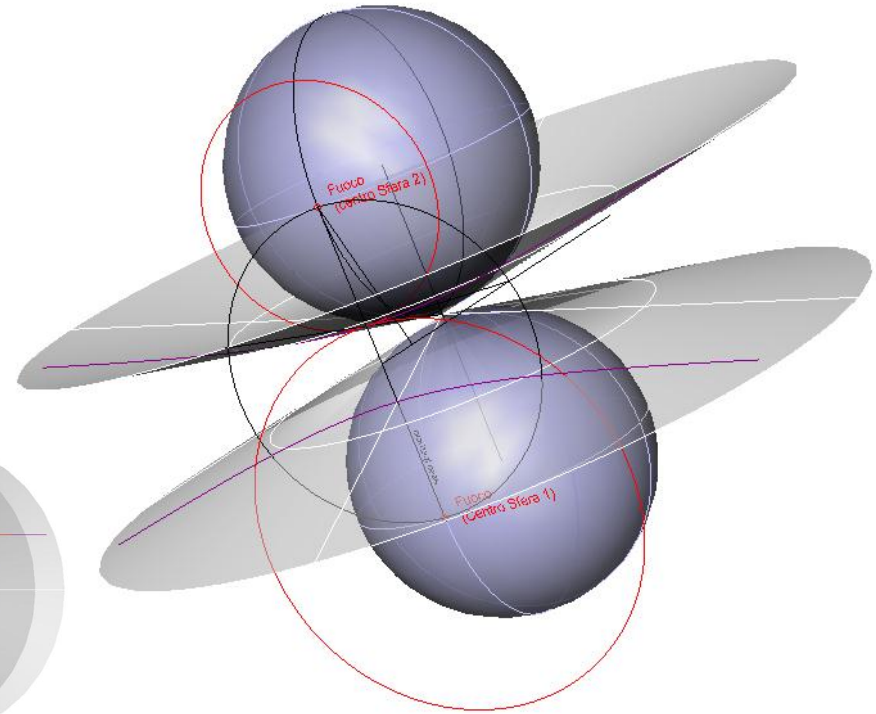


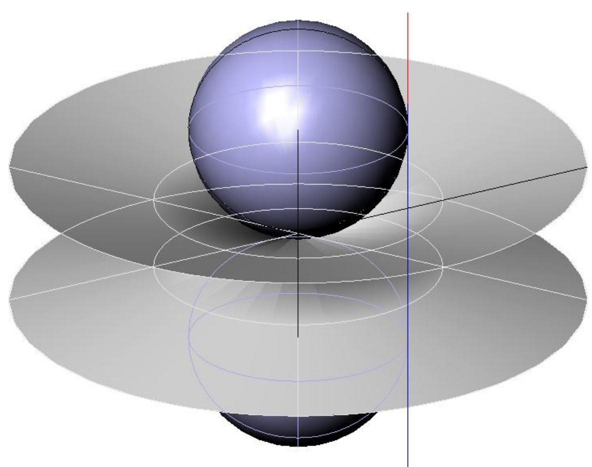
Le **sfere** di **Dandelin**

- caso dell’**iperbole**

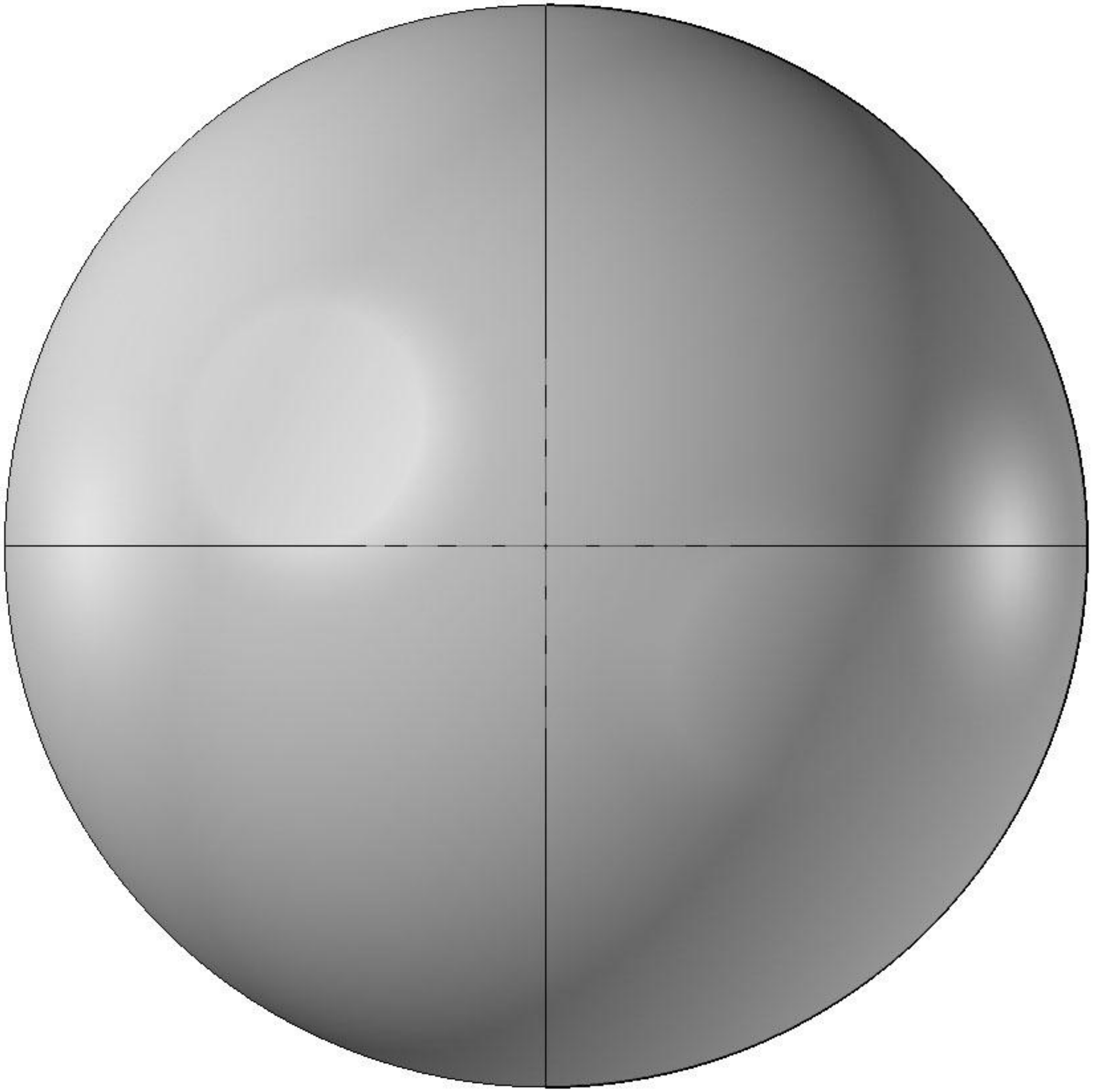




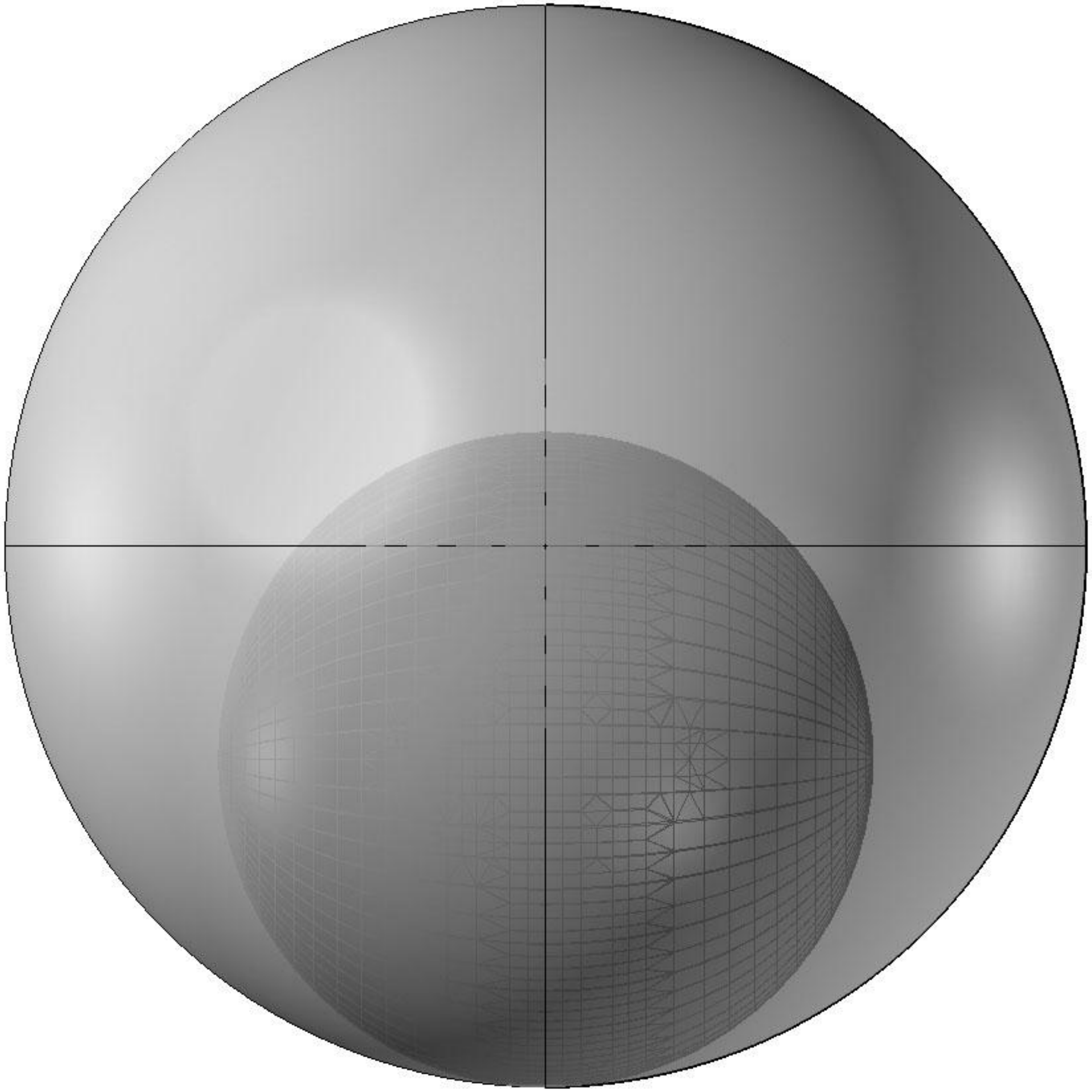




sfera-madre - **sfera 1**



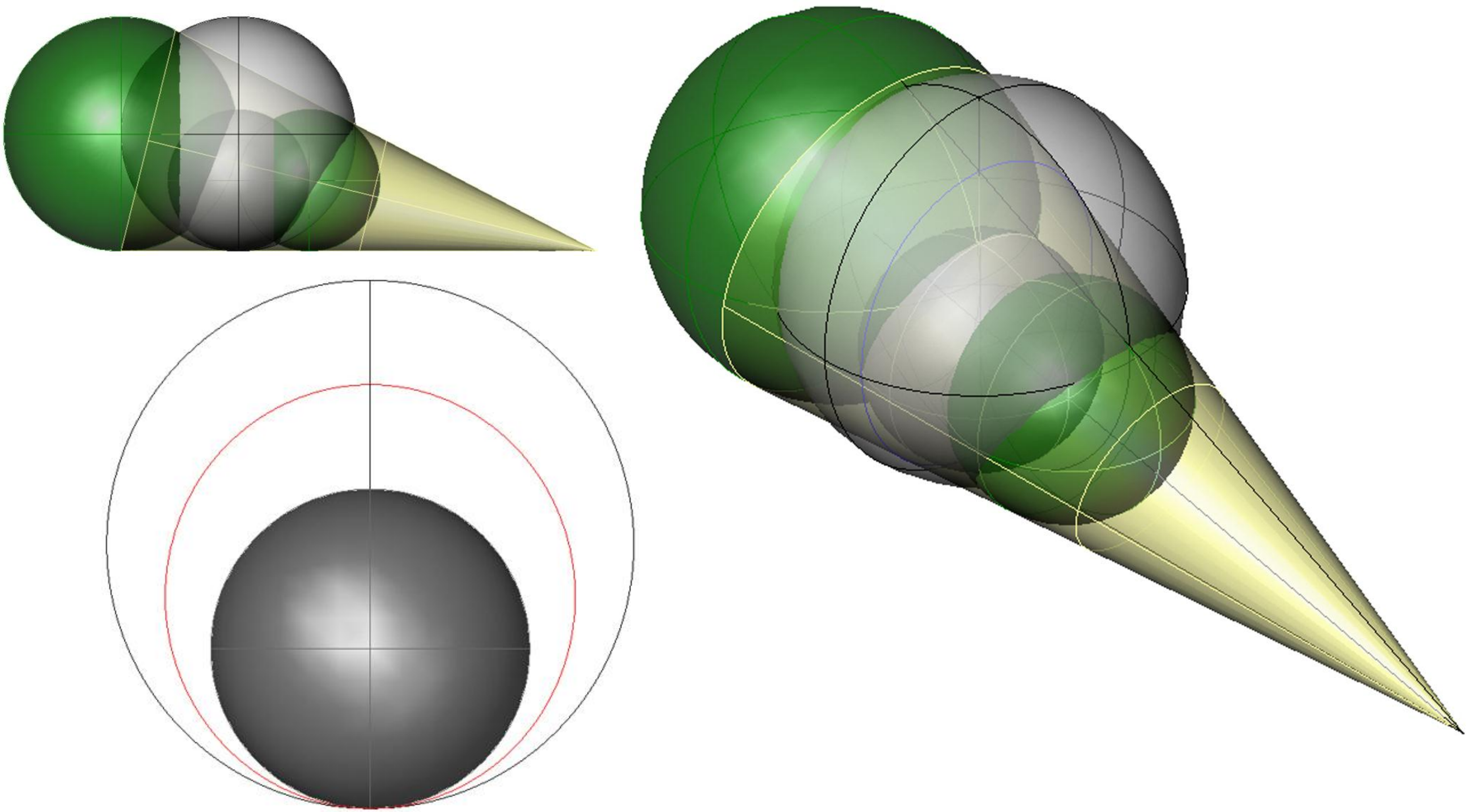
sfera 2



L'anonimo, Soddy, il teorema

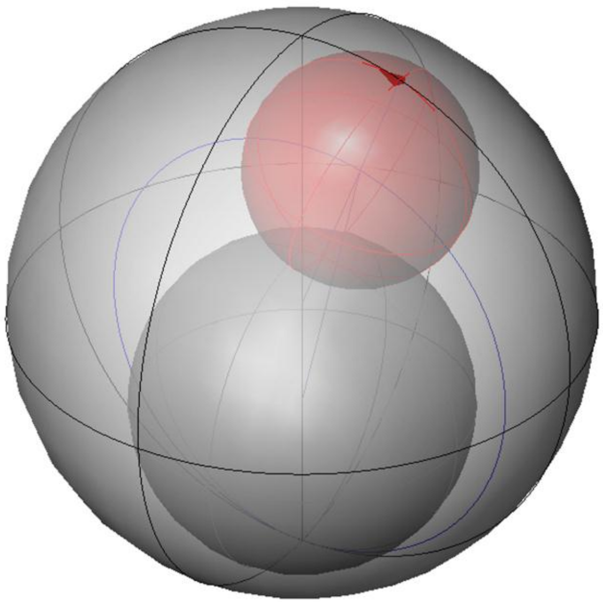
Sfere di Dandelin per **ellisse**,

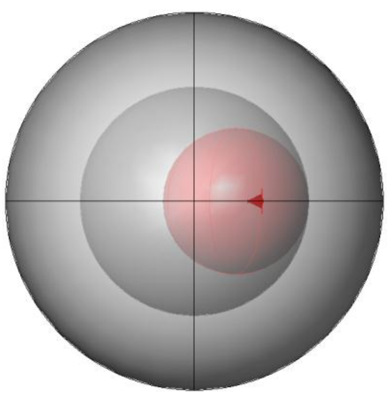
specifico luogo di equidistanza tra la **sfera 1** e la **sfera 2**

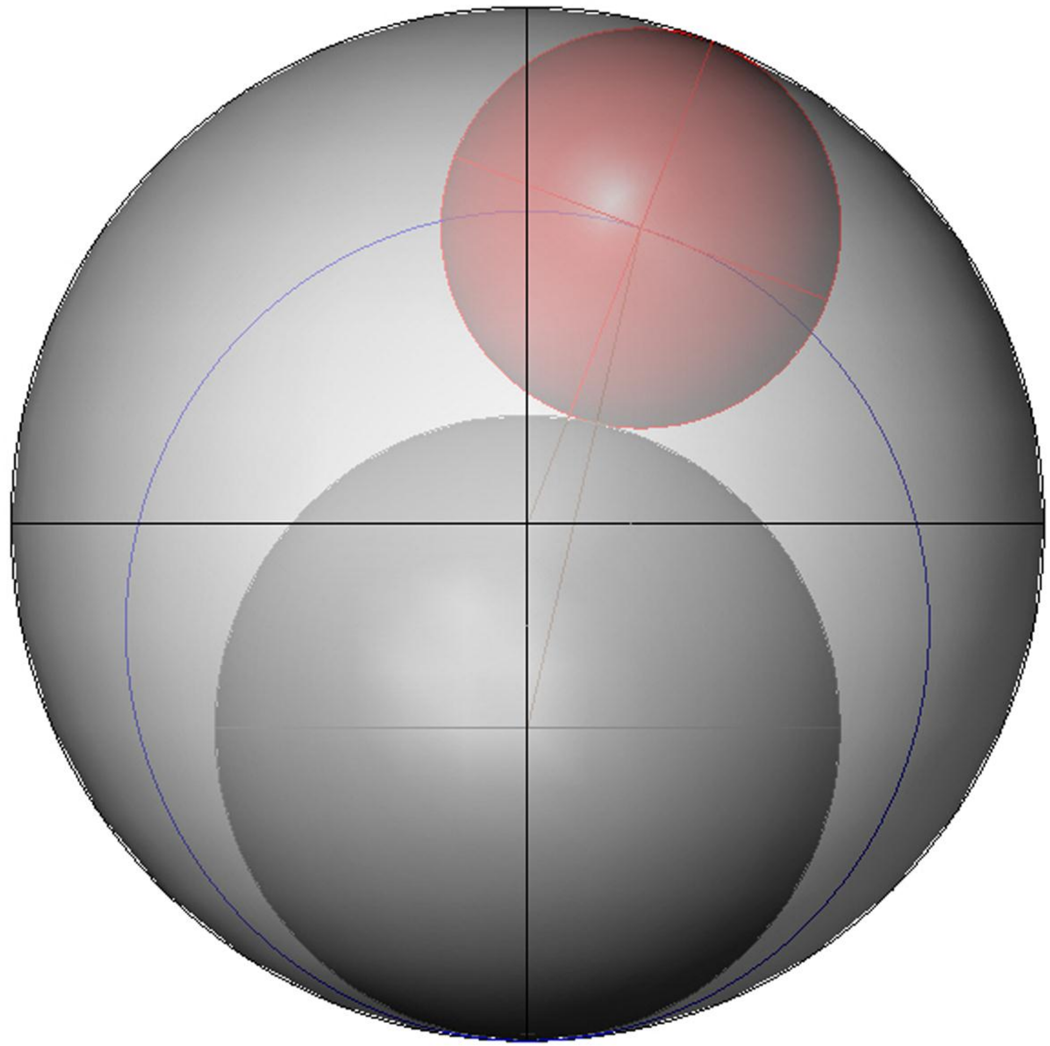


L’anonimo, Soddy, il teorema

costruzione di una terza sfera tangente alle precedenti due - **sfera 3**



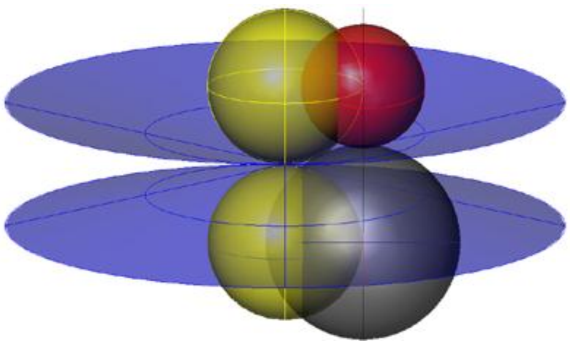


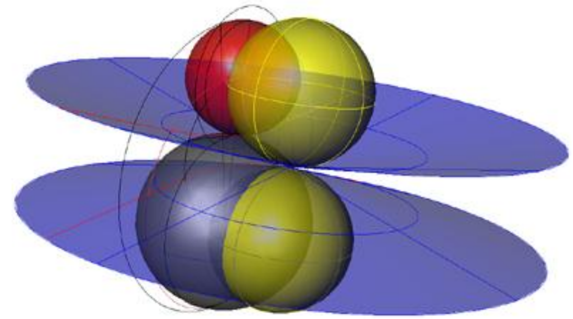


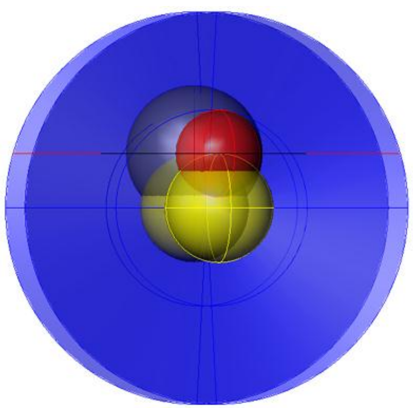
L’anonimo, Soddy, il teorema

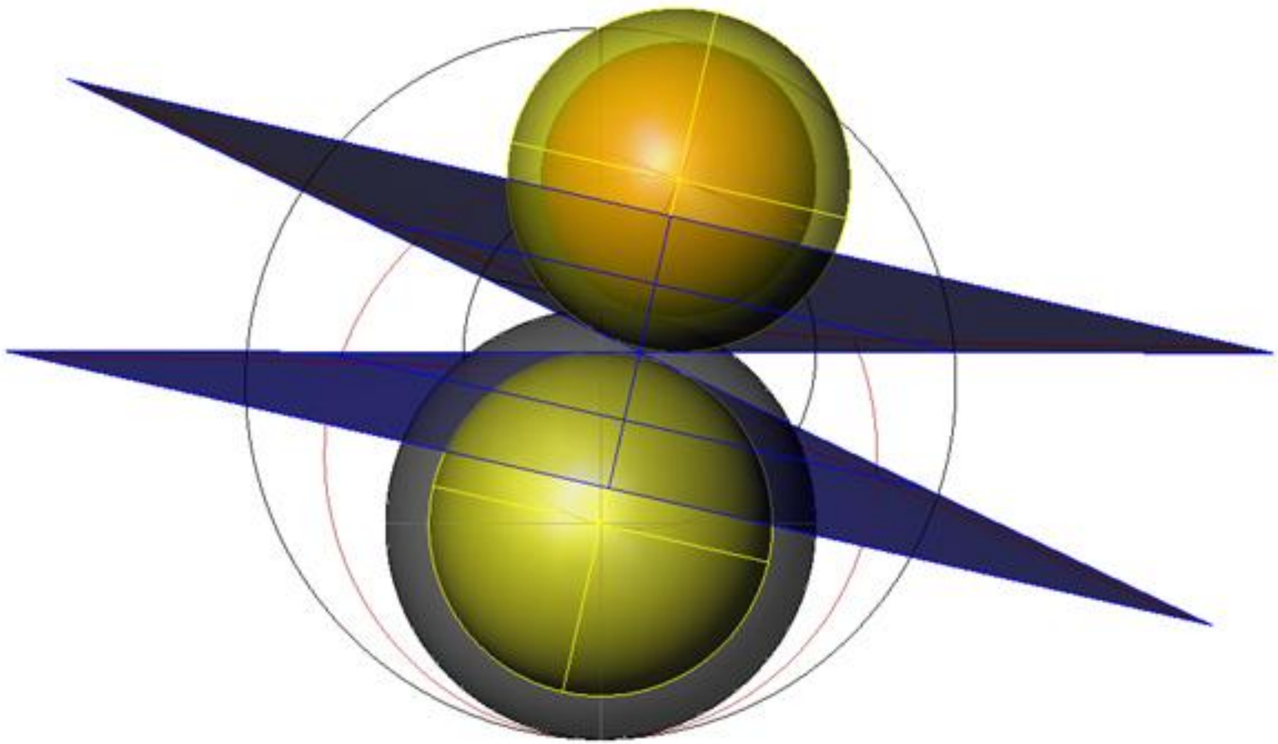
Sfere di Dandelin per **iperbole 1**,

specifico luogo di equidistanza tra la **sfera 2** e la **sfera 3**

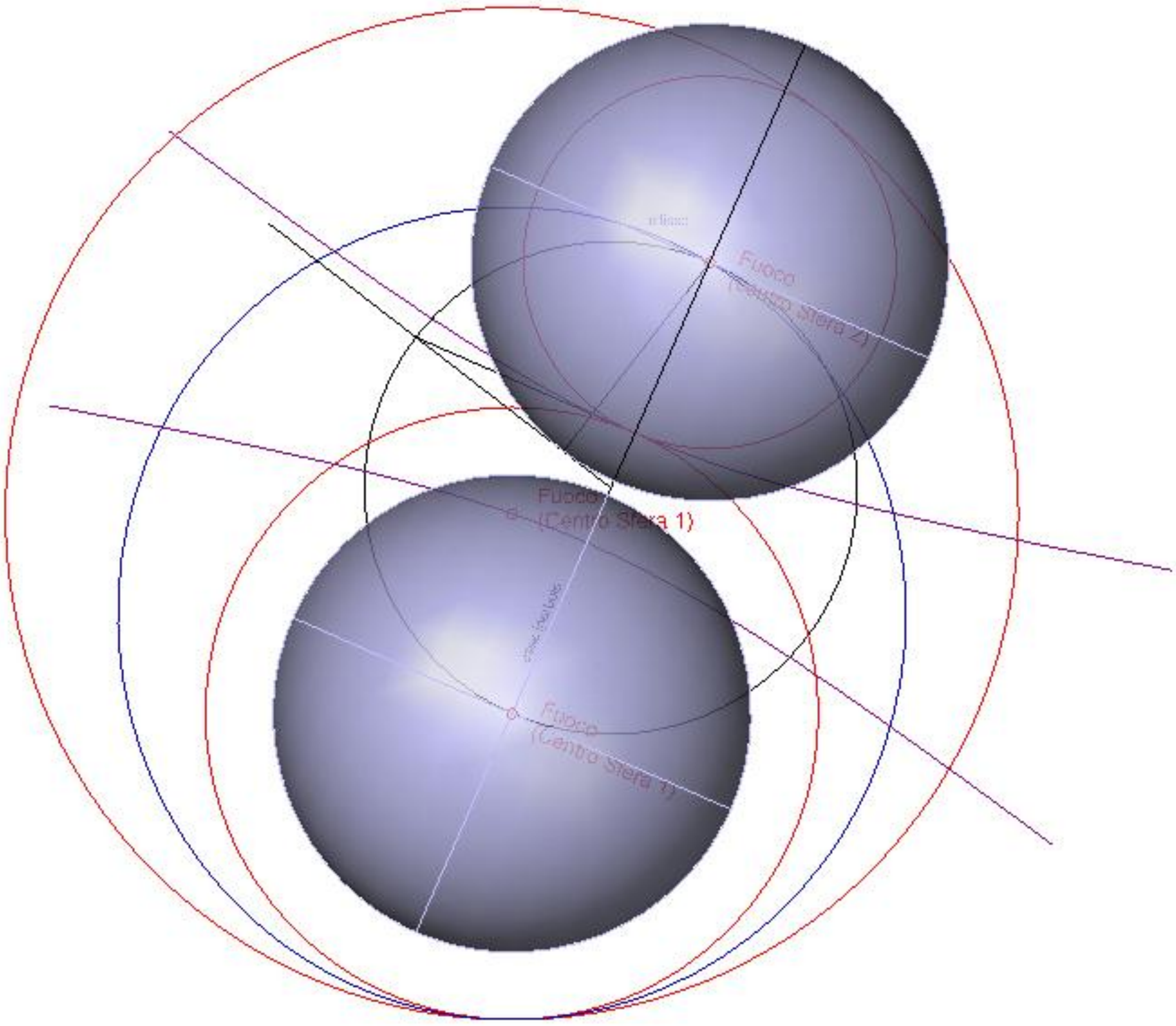




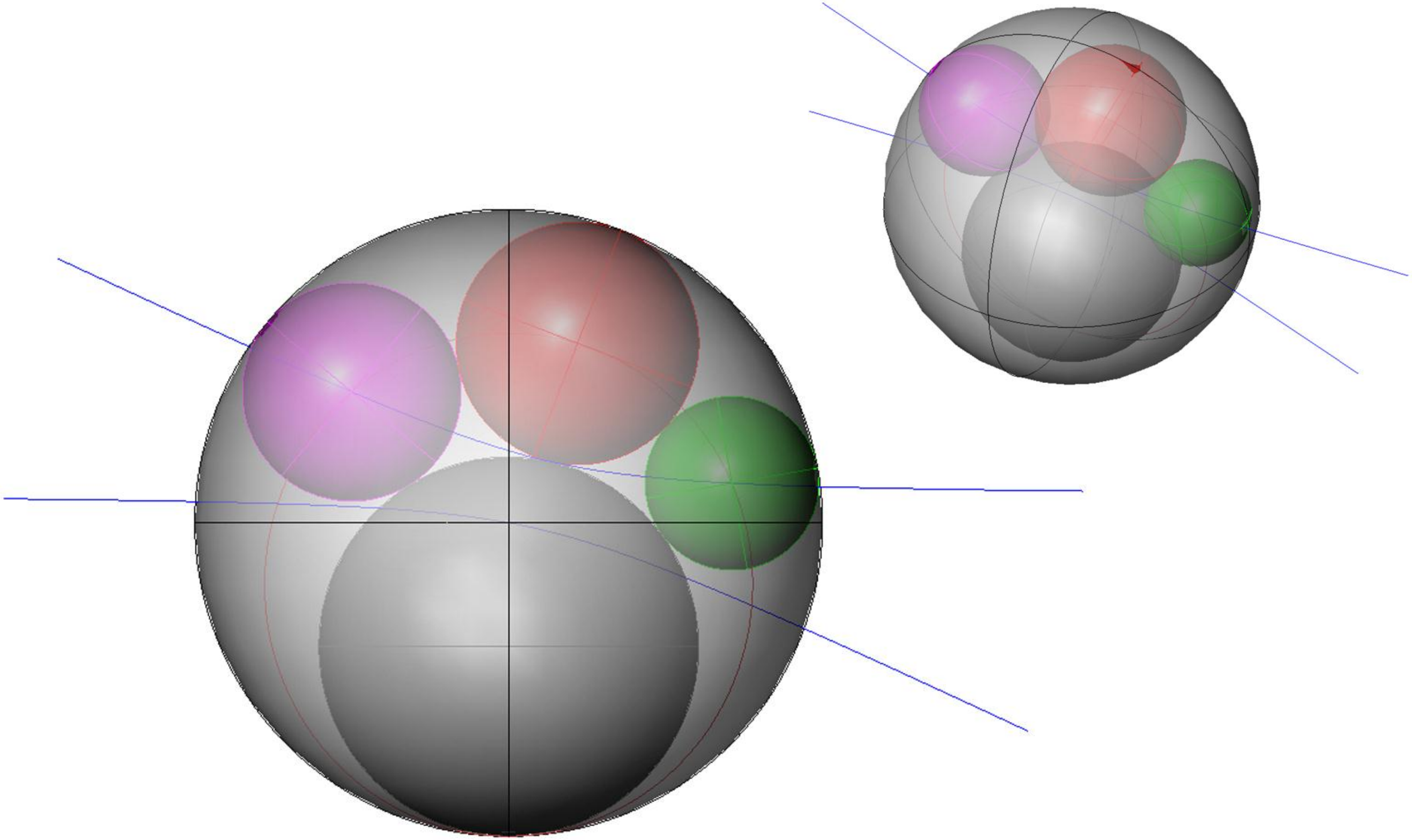




iperbole 1 luogo di equidistanza tra la **sfera 2** e **sfera 3**



sfera 4 e **sfera 5**



Fino alla sfera **5** si è potuto operare utilizzando oggetti bidimensionali (intersezione tra rette, cerchi, coniche).

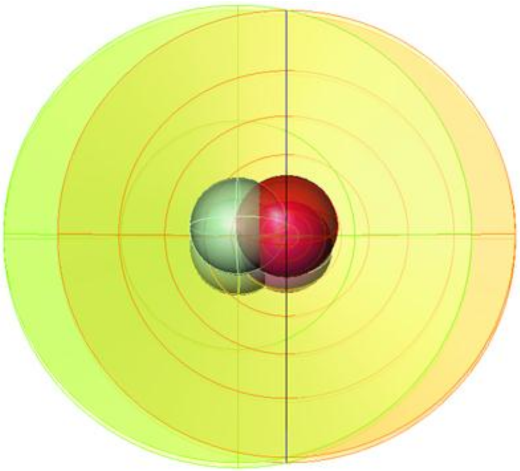
Per proseguire occorre utilizzare oggetti tridimensionali (intersezione tra piani, sfere, solidi di rotazione generati da coniche, come ellissoidi ed iperboloidi).

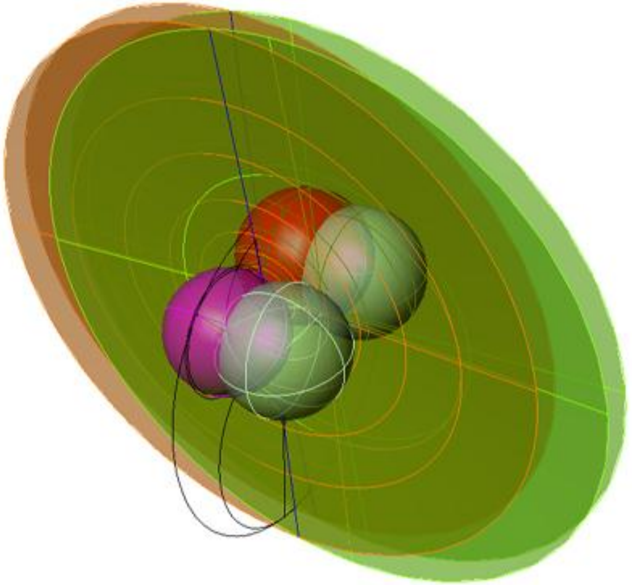
Da qui in poi, per determinare un punto nello spazio non sono più sufficienti due oggetti, ma ne occorrono tre.

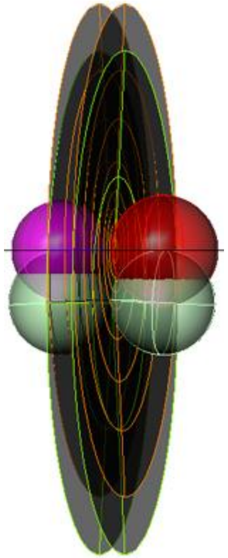
L’anonimo, Soddy, il teorema

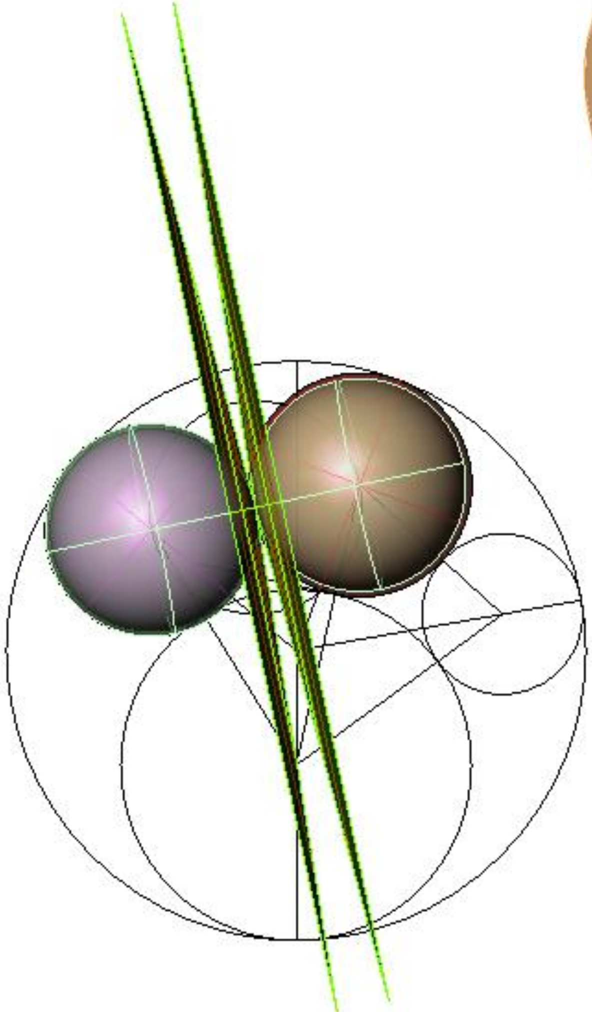
Sfere di Dandelin per **iperbole 2**,

specifico luogo di equidistanza tra la **sfera 3** e la **sfera 4**

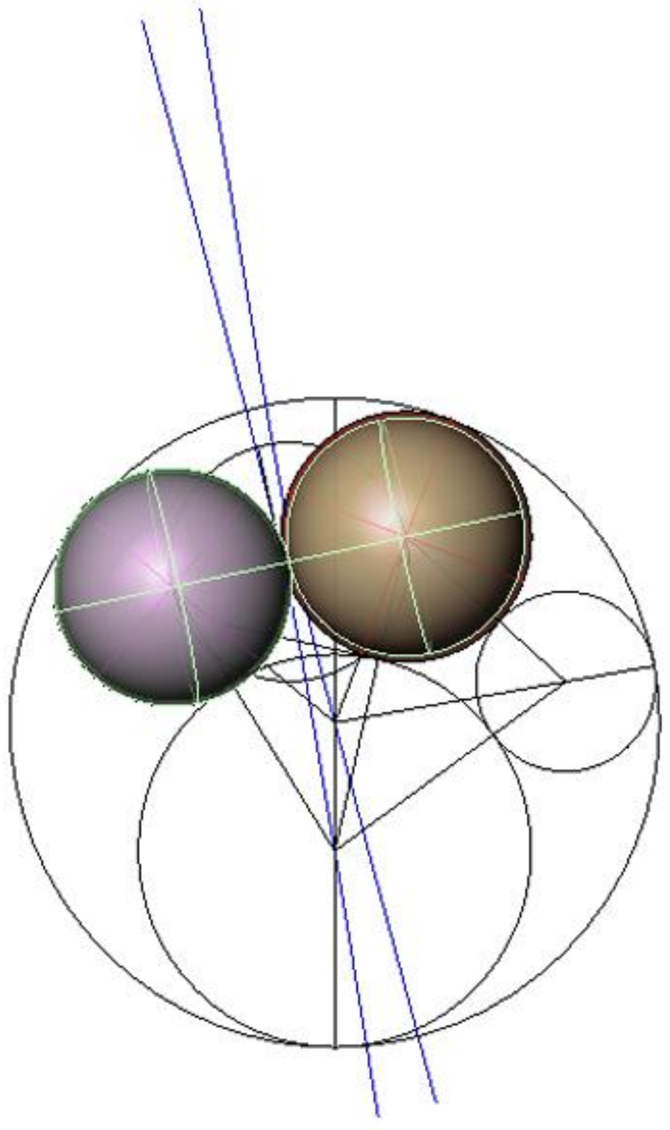








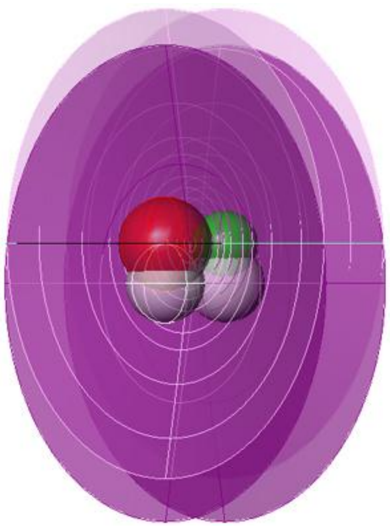
iperbole 2, luogo di equidistanza tra la sfera **3** e la sfera **4**

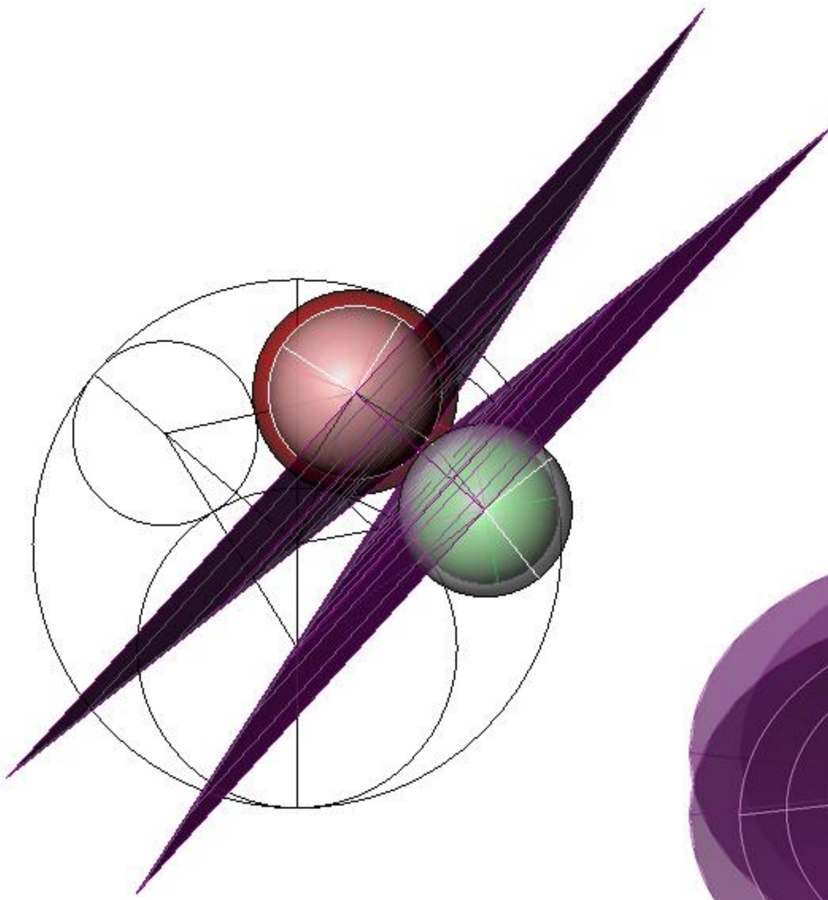


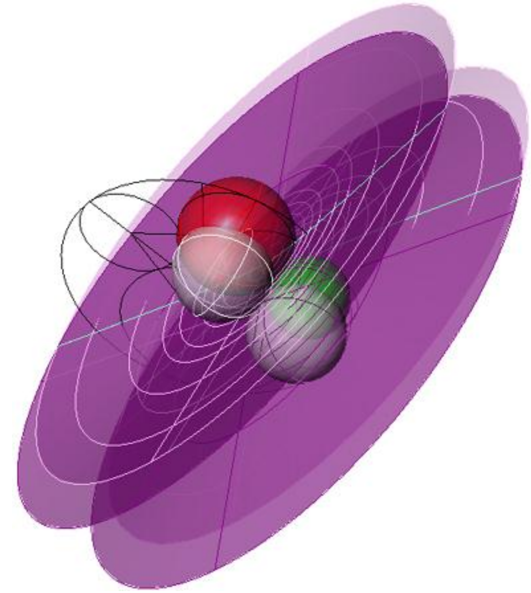
L’anonimo, Soddy, il teorema

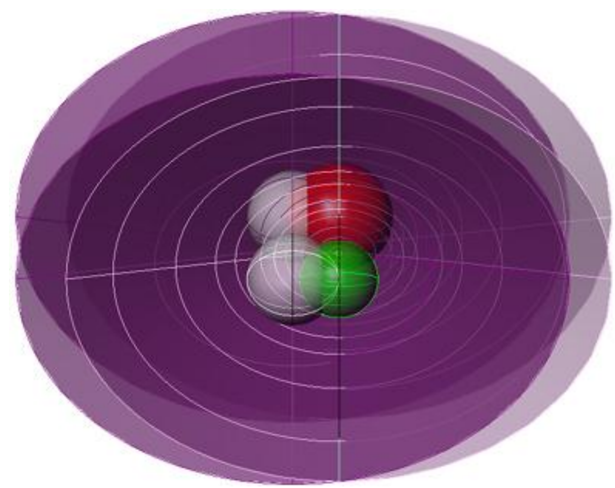
Sfere di Dandelin per **iperbole 3**,

specifico luogo di equidistanza tra la **sfera 4** e la **sfera 5**

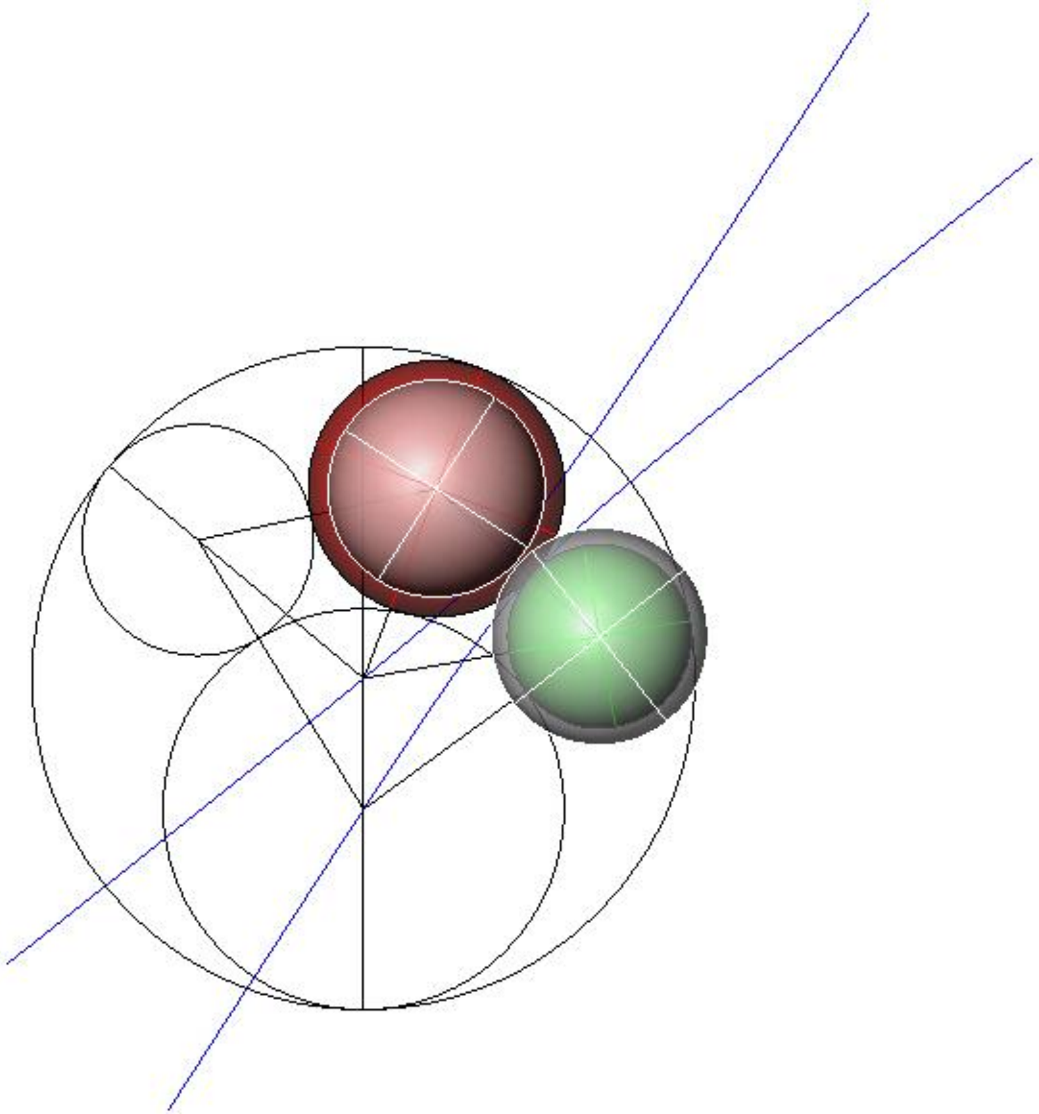




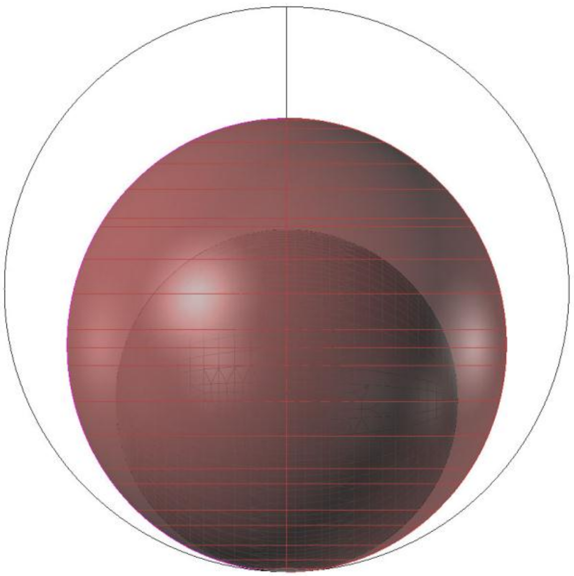


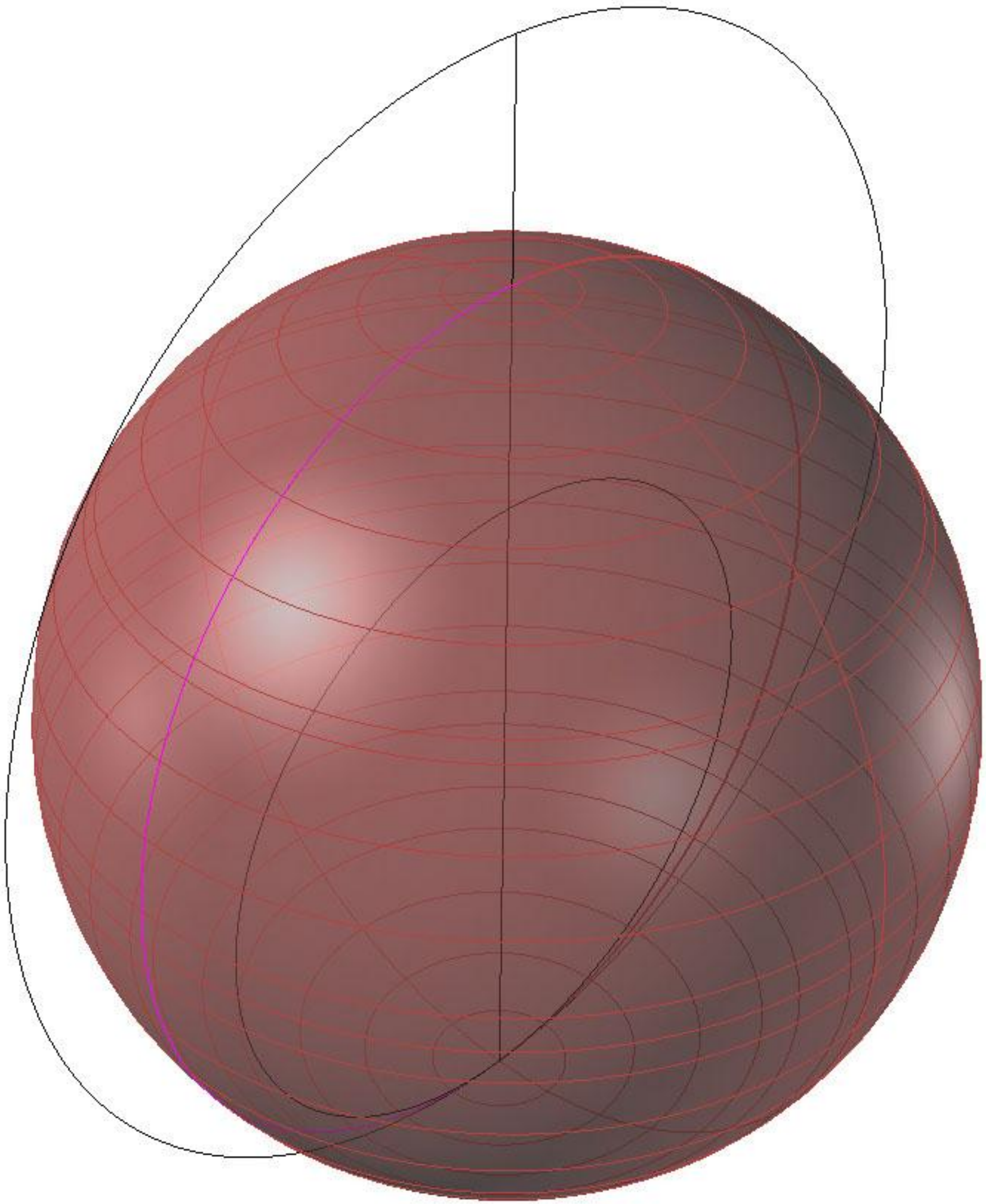


iperbole 3, luogo di equidistanza tra la **sfera 3** la **sfera 5**

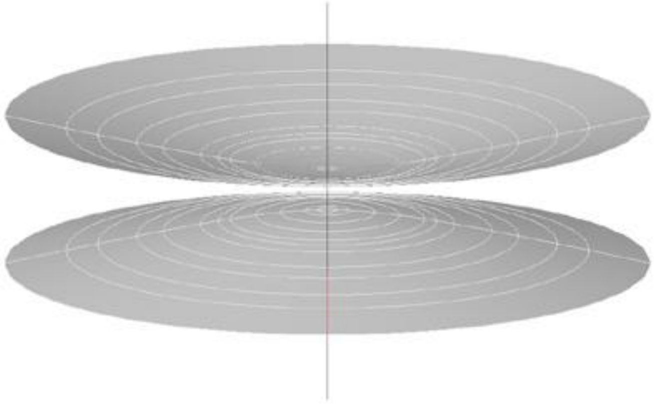


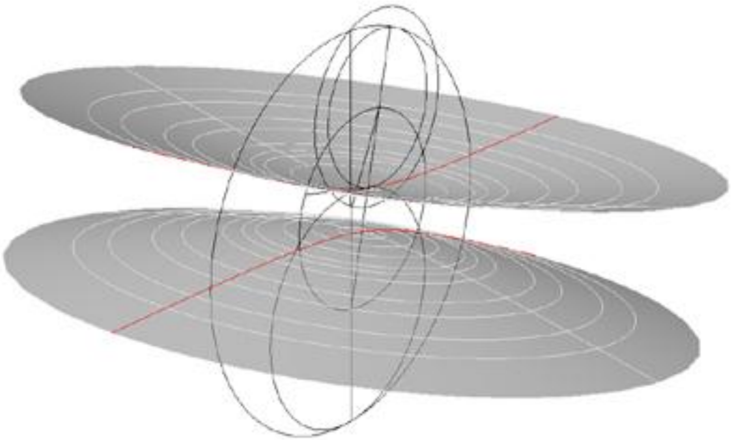
ellisse (**ellissoide**)

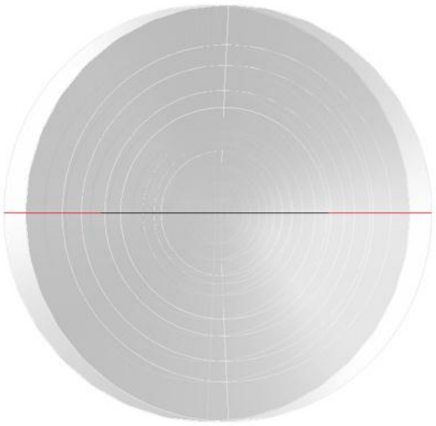


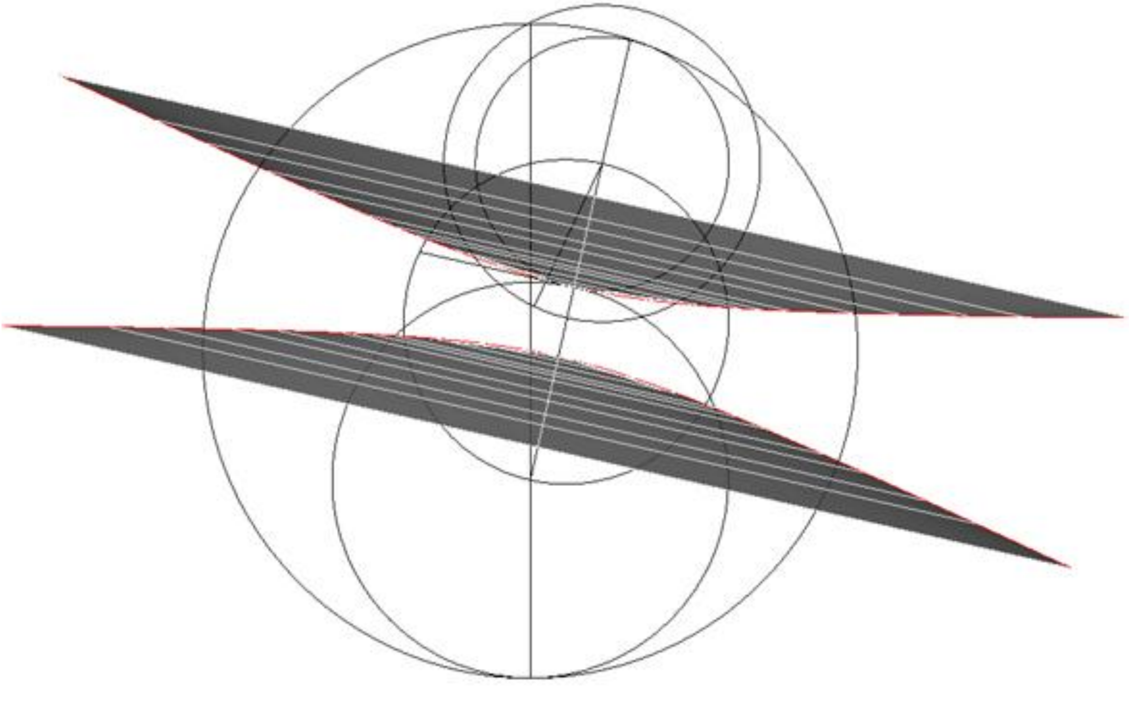


iperbole **1** (**iperboloide 1**)

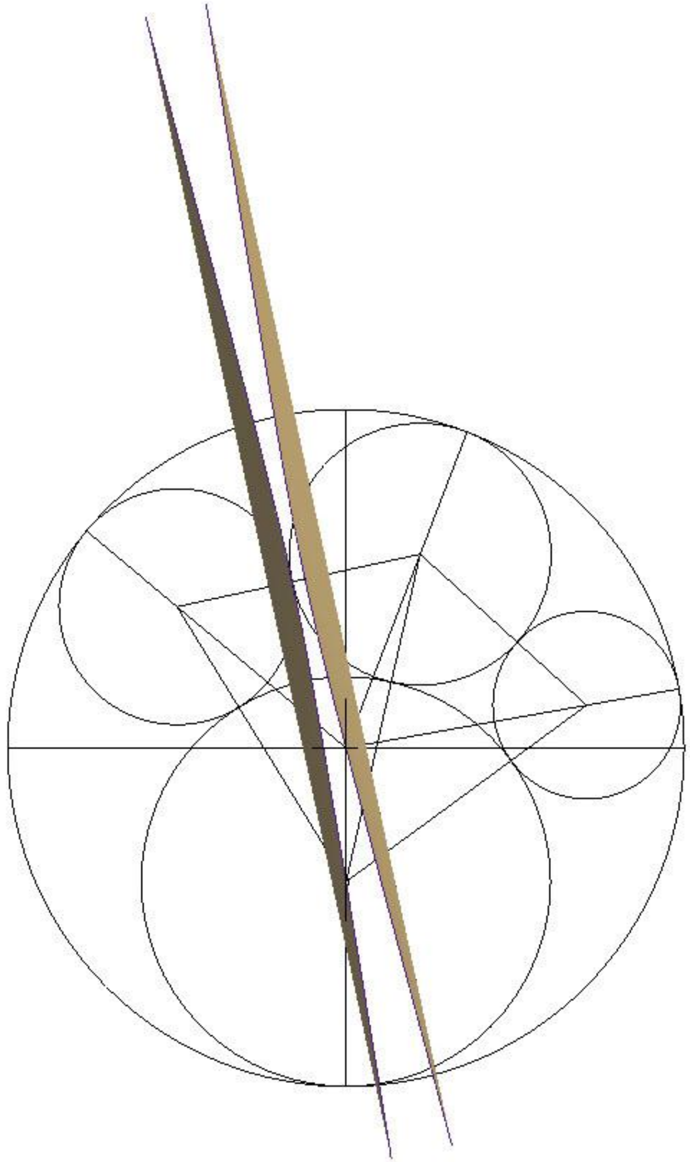


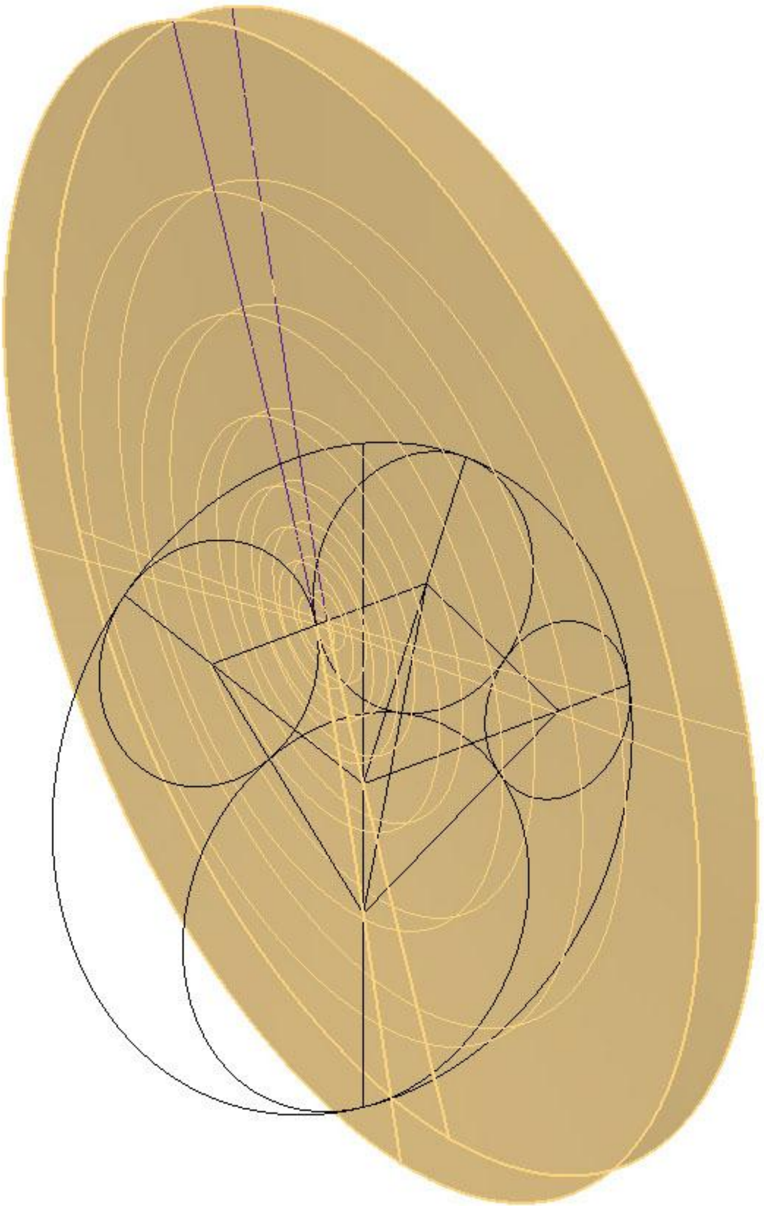




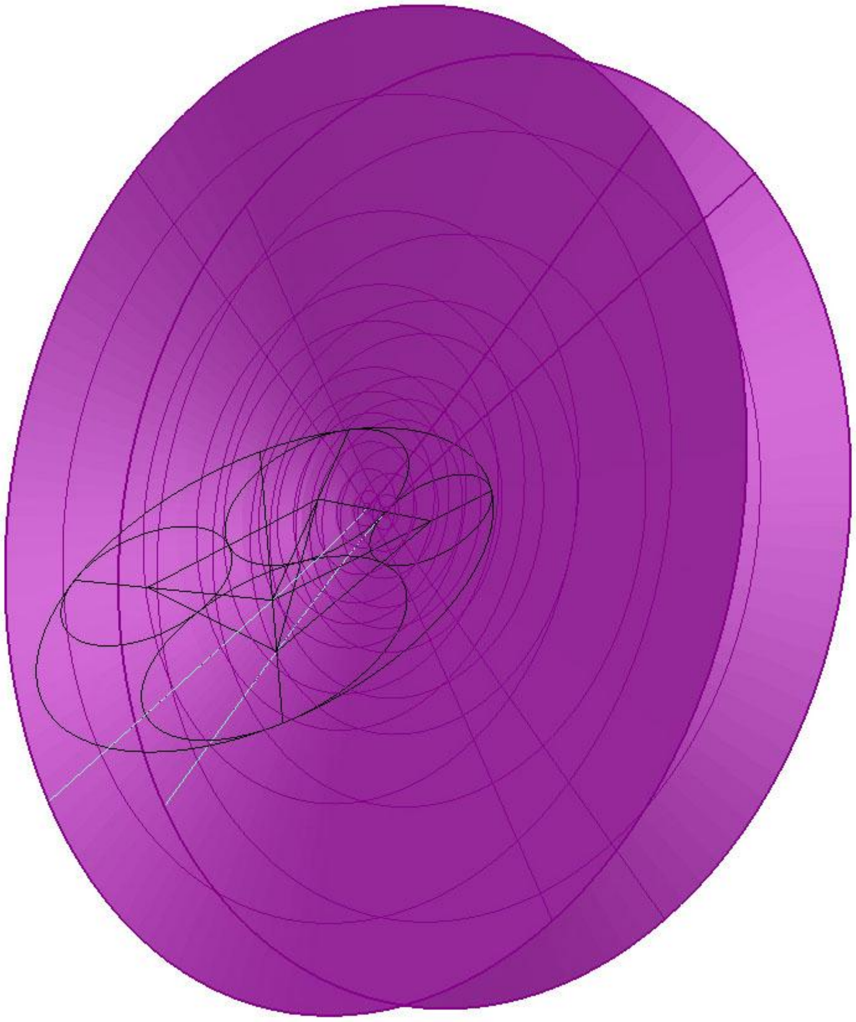


iperbole **2** (**iperboloide 2**)

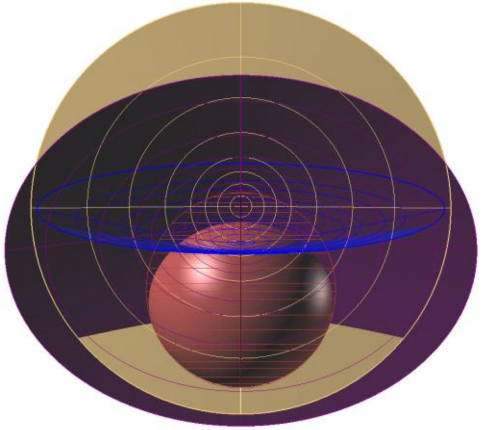


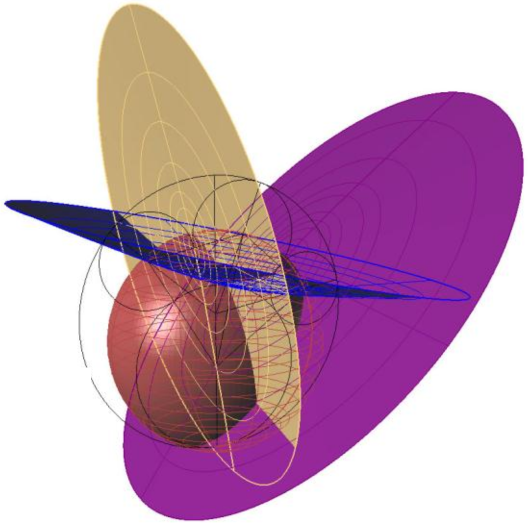


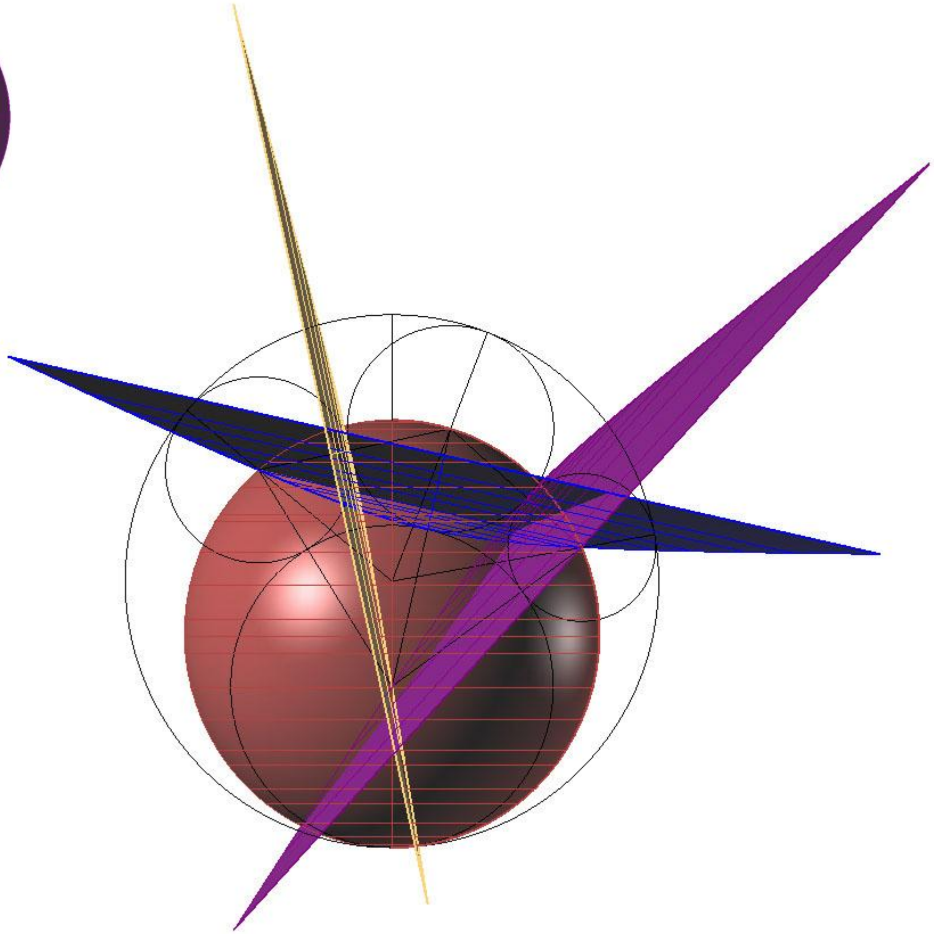
iperbole **3** (**iperboloide 3**)



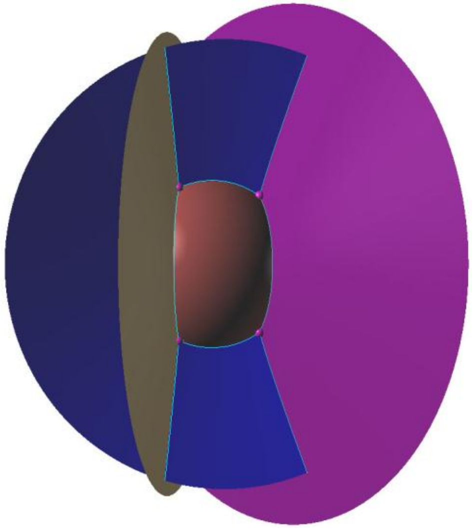
Oggetti 3D raggruppati insieme

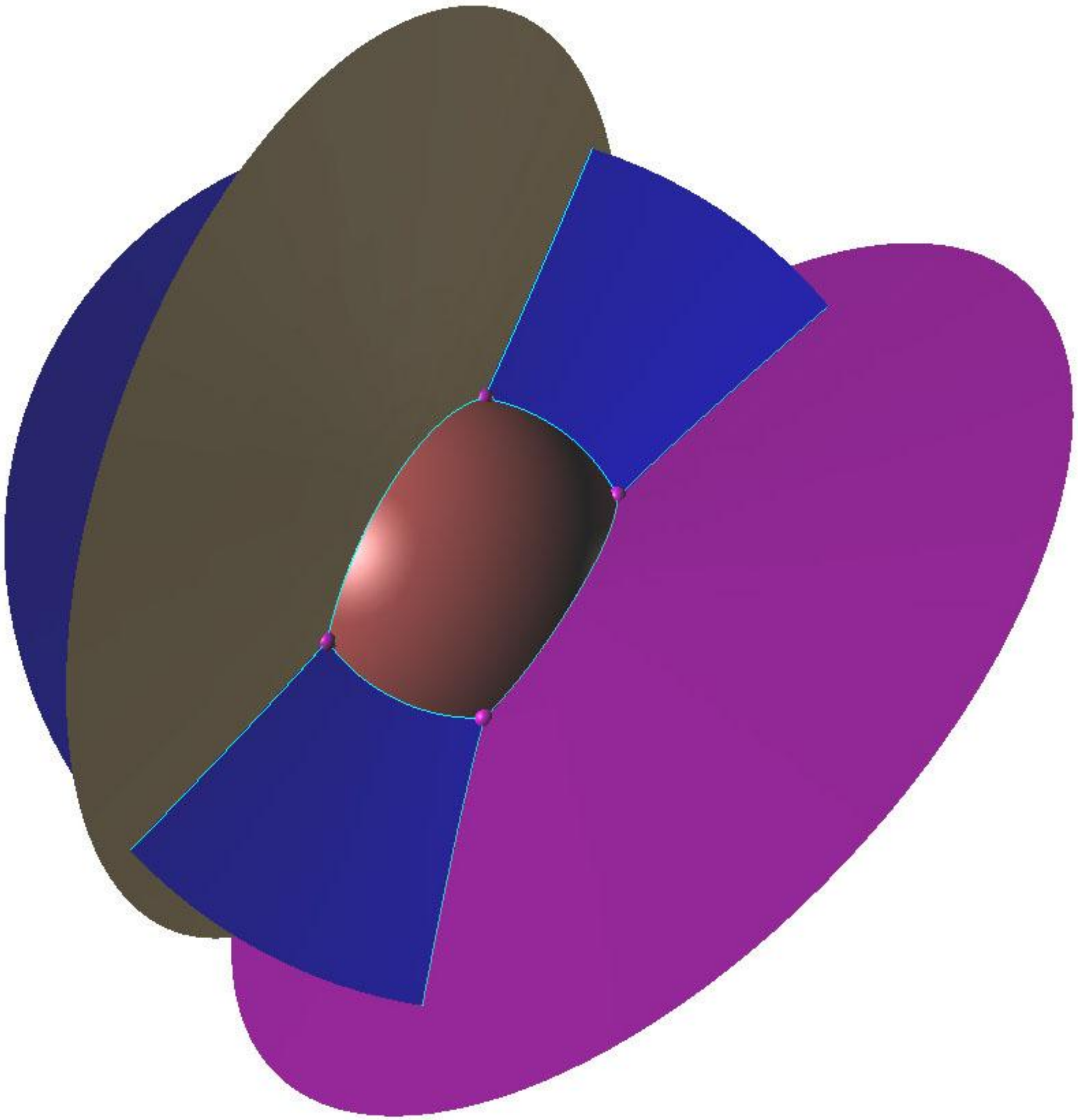




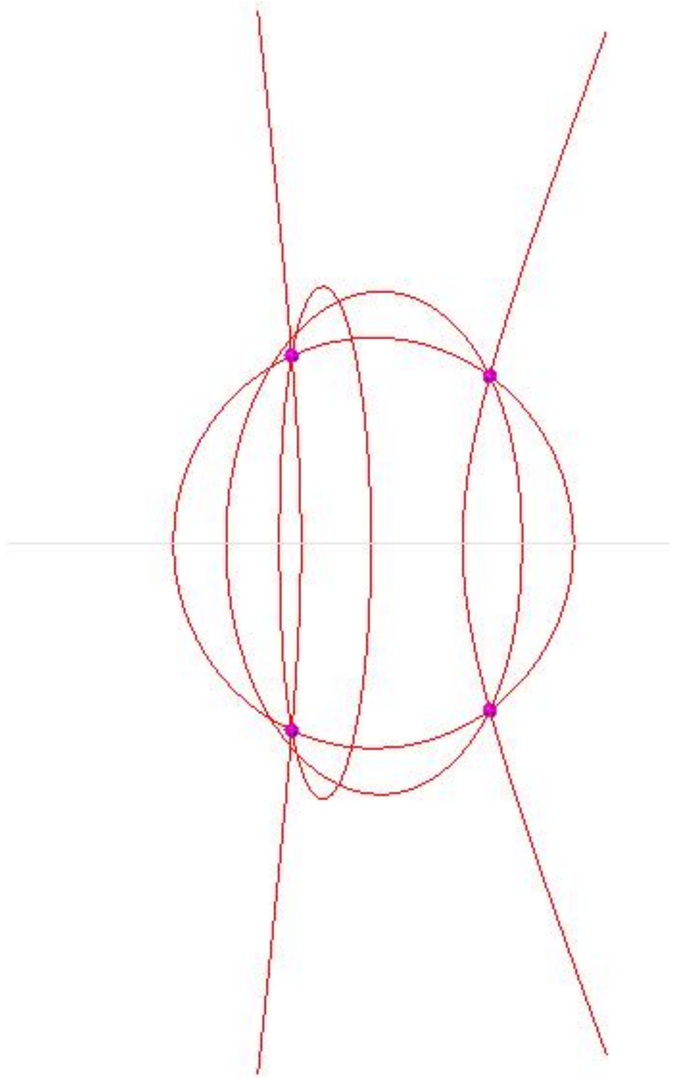


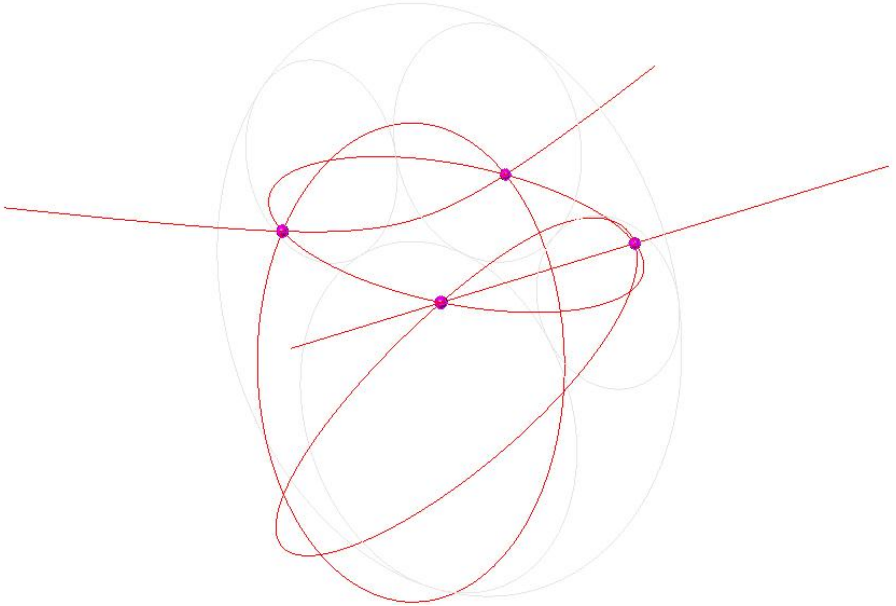
linee-intersezioni tra gli oggetti 3D



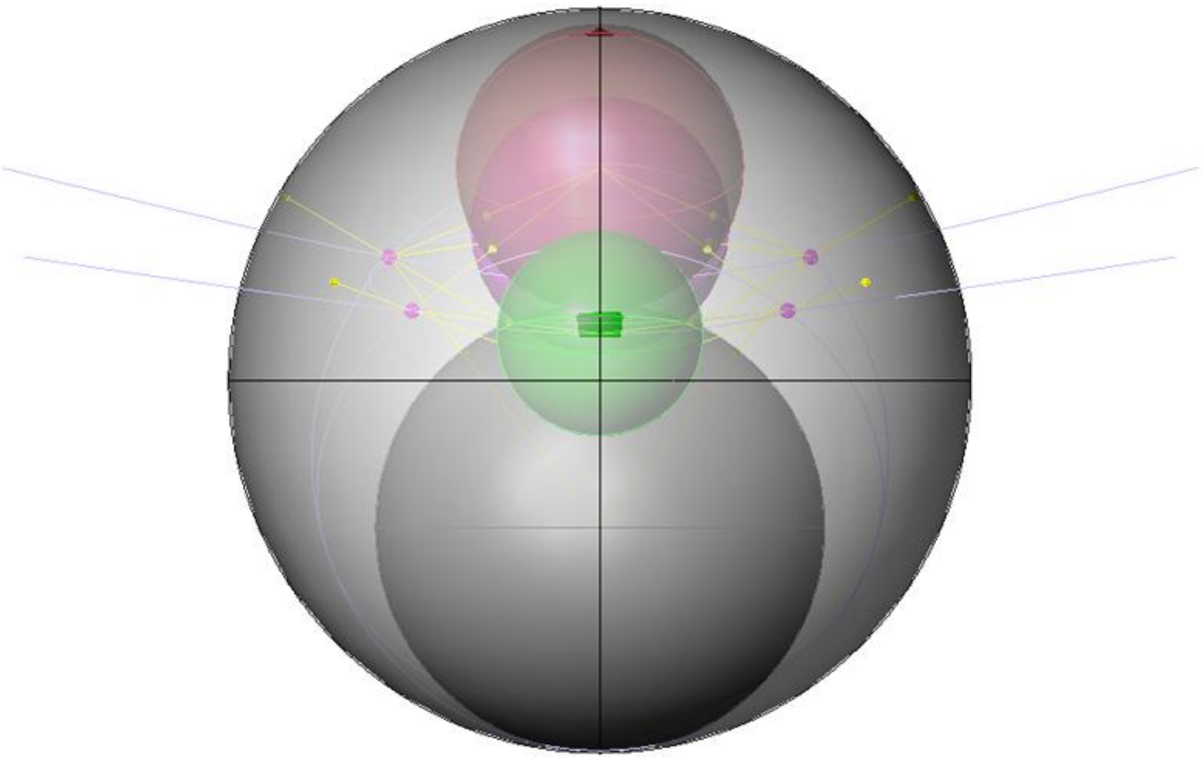


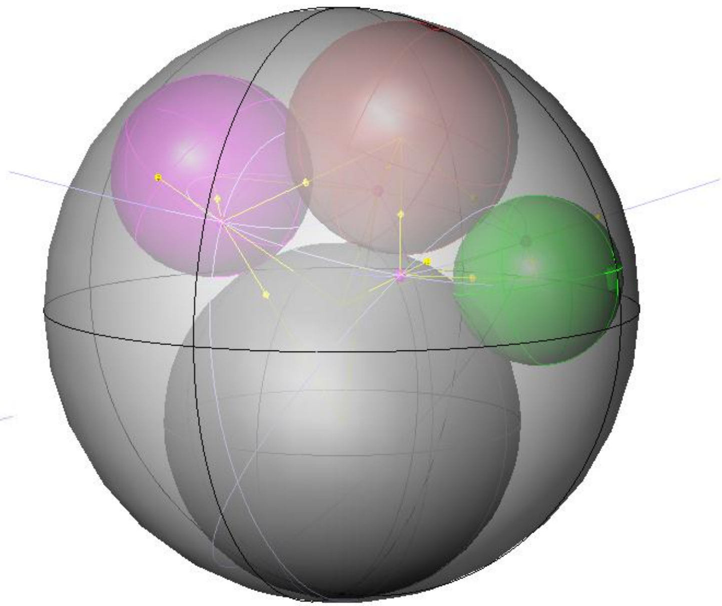
I quatto punti di intersezione, distinti secondo due coppie simmetriche





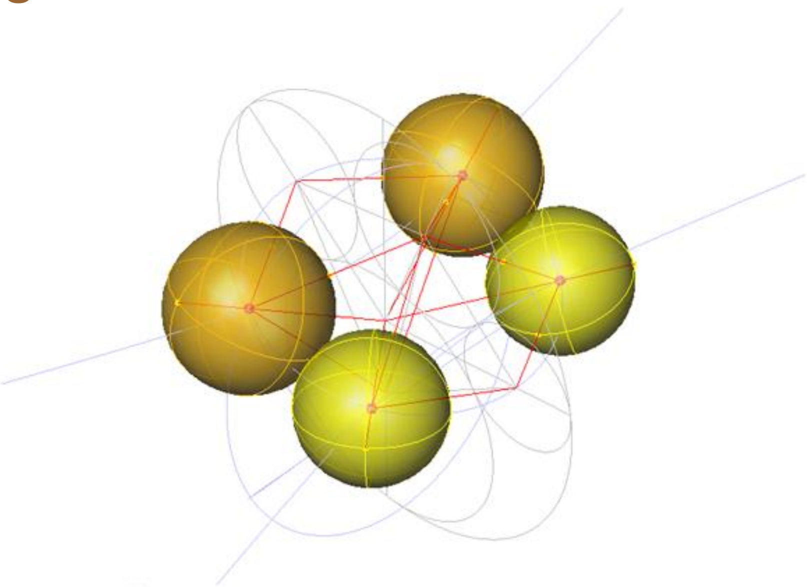
I raggi delle quattro sfere che mancano

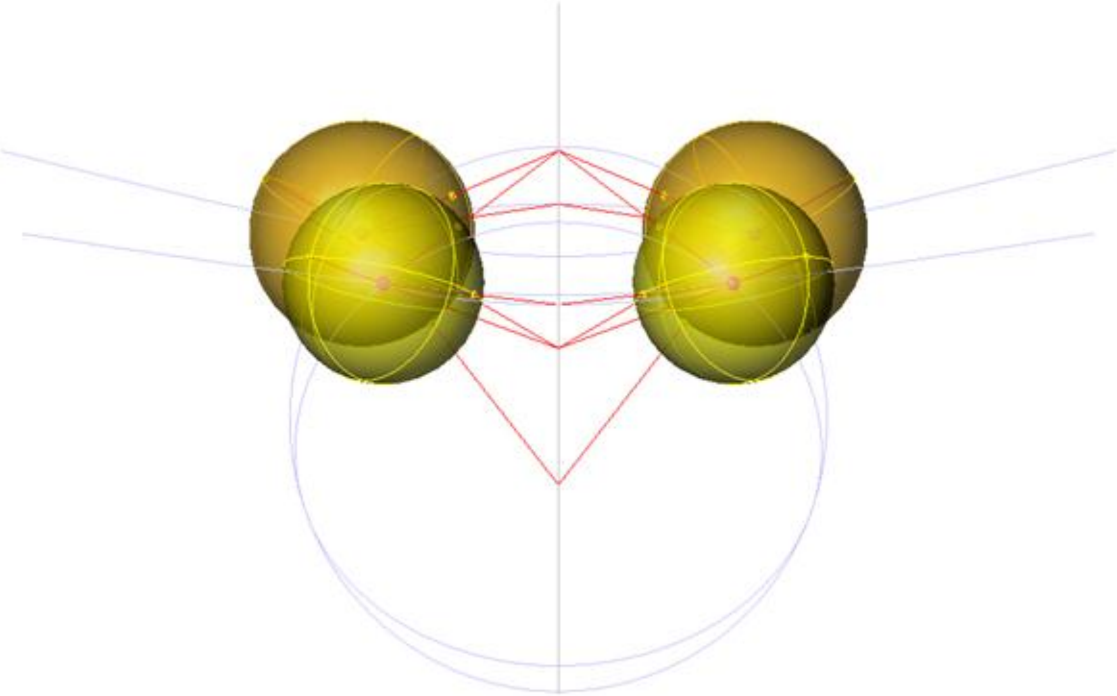




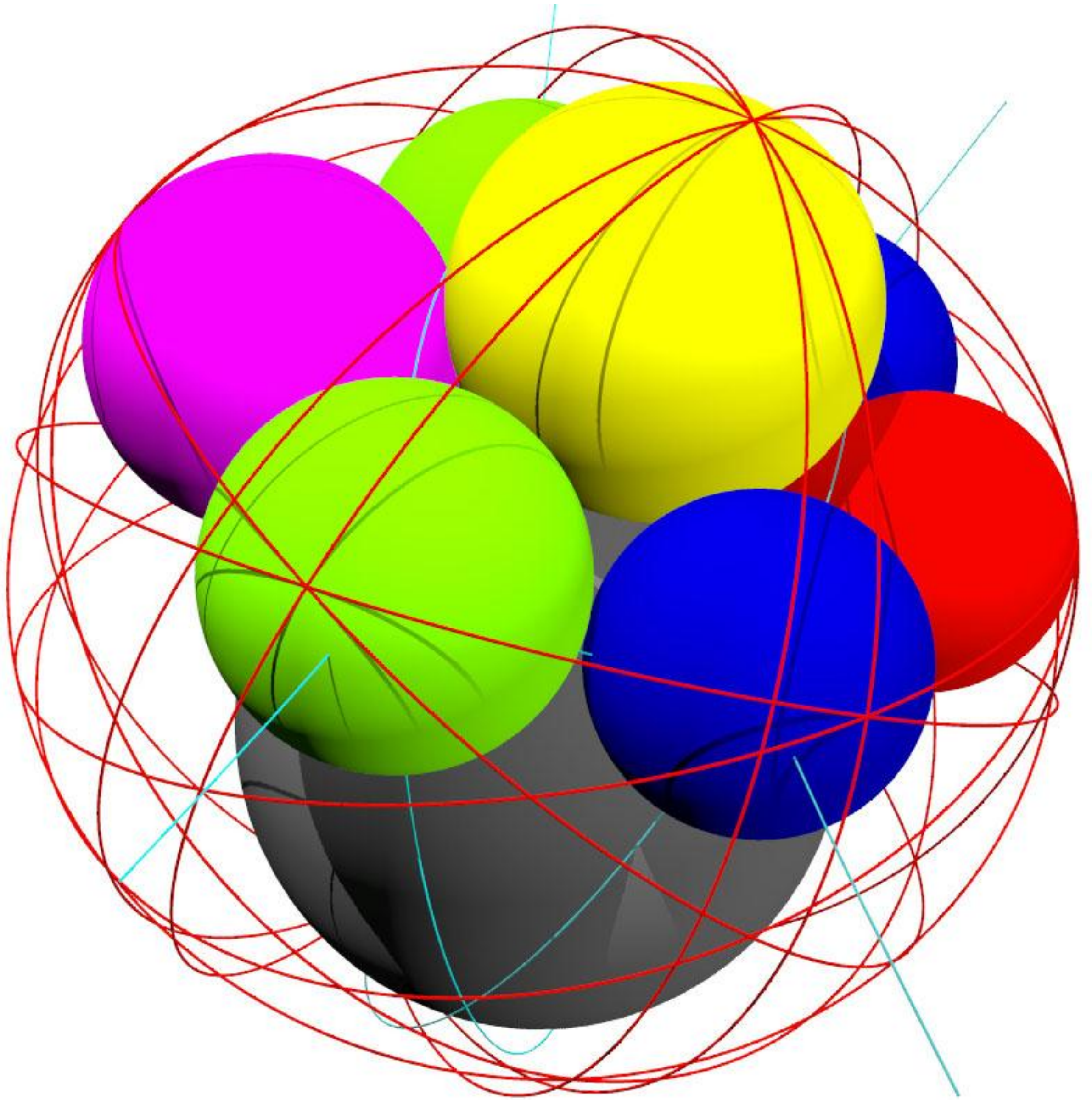
Quattro sfere, rispettivamente tangenti alle sfere:

1; 2; 3; **4** e 1; 2; 3; **5**

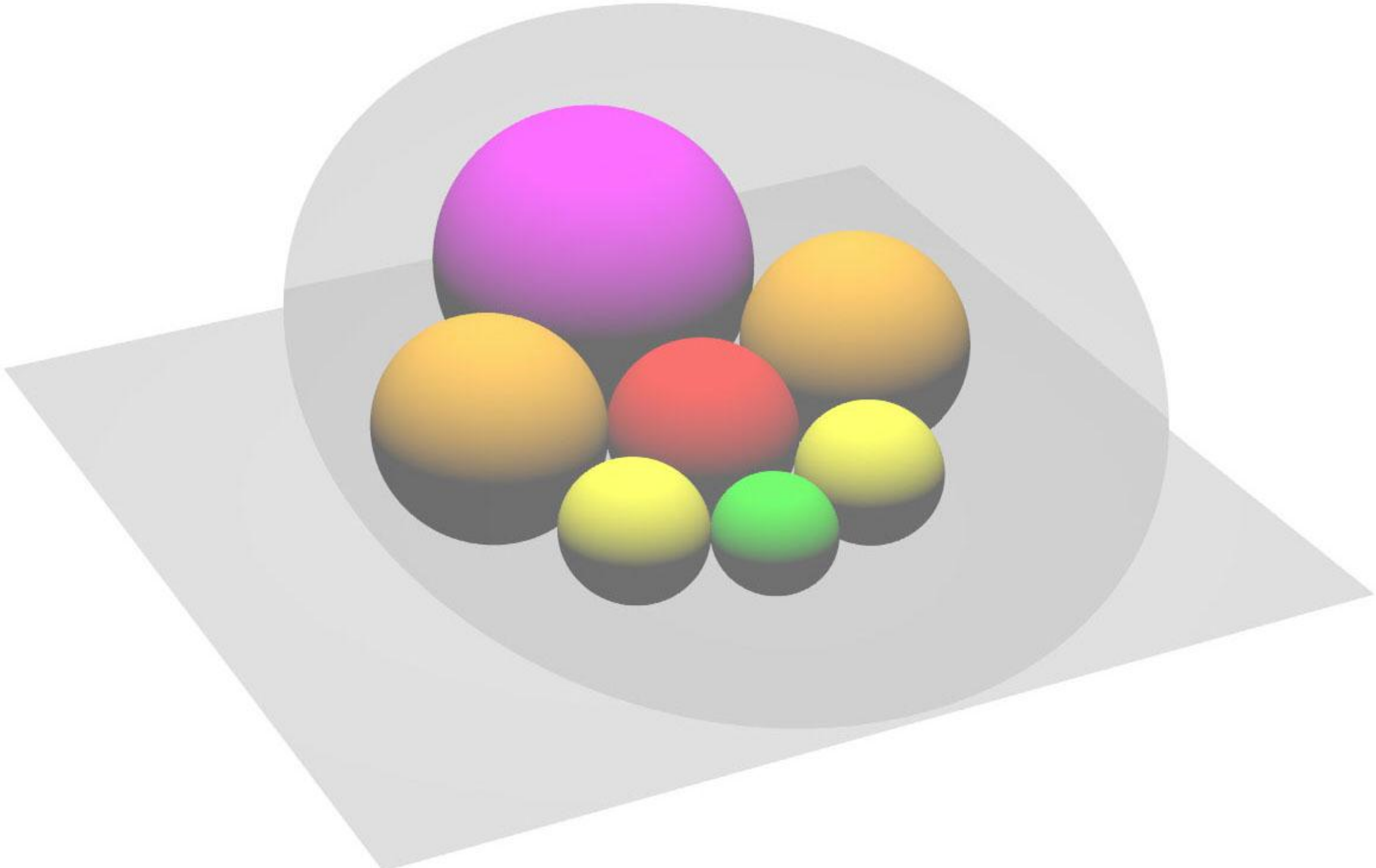




Nove sfere si baciano

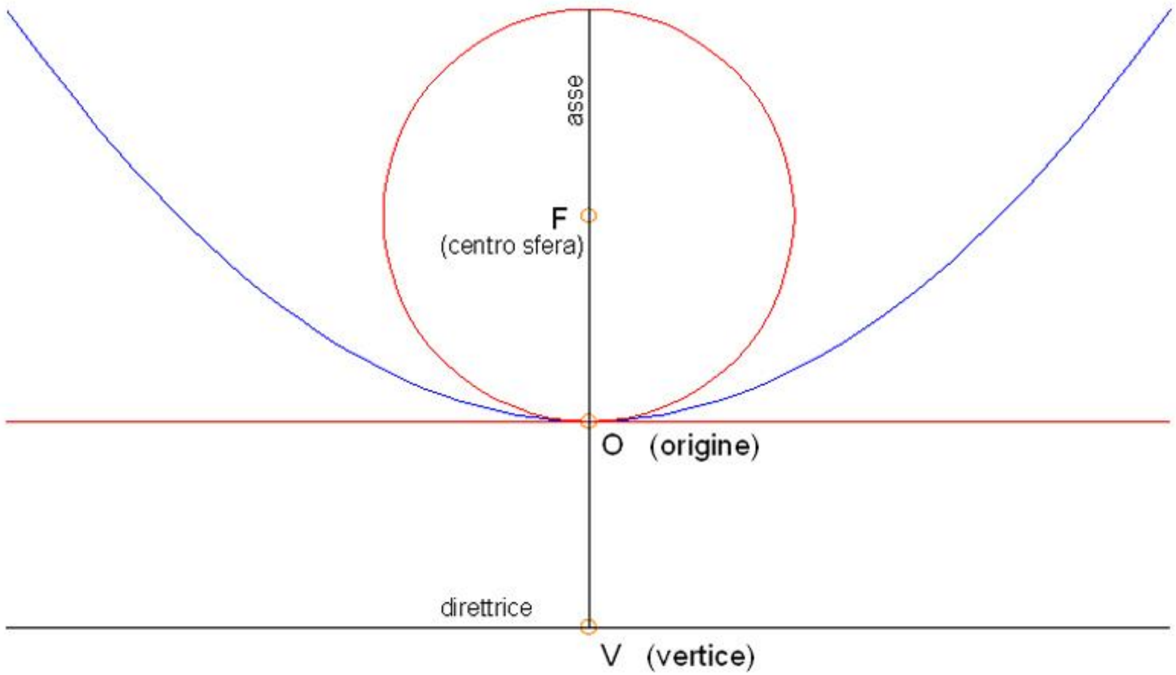


Se la **sfera 1** si trasforma in un **piano** (raggio infinito)



Il luogo di equidistanza tra retta e cerchio tangenti è una **parabola** dalle seguenti proprietà:

- il **fuoco** coincide con il **centro** del cerchio
- l’**origine** coincide con la **retta** tangente



L’anonimo, Soddy, il teorema

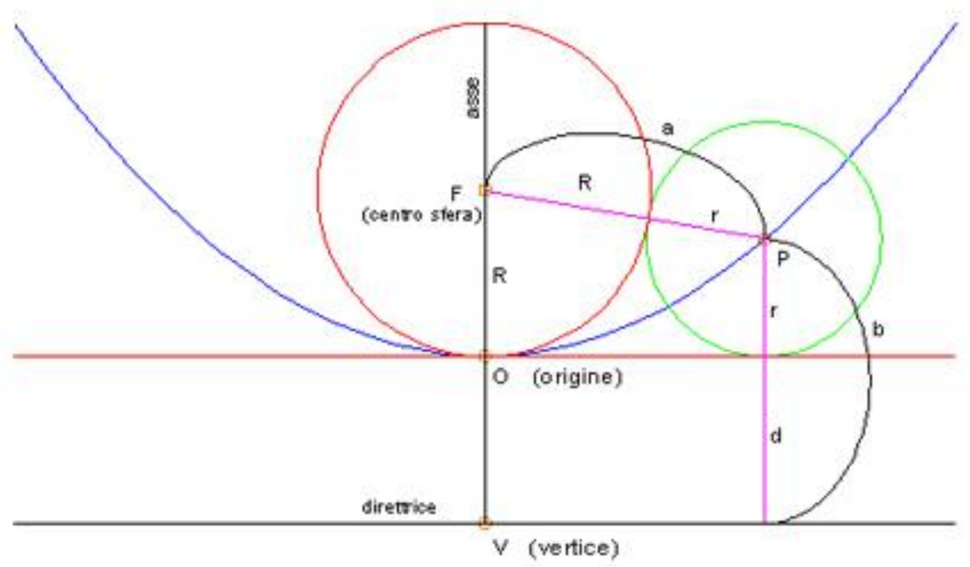
In ogni parabola **a = b** **a** (distanza **F-P**) **b** (distanza **P-direttrice**)

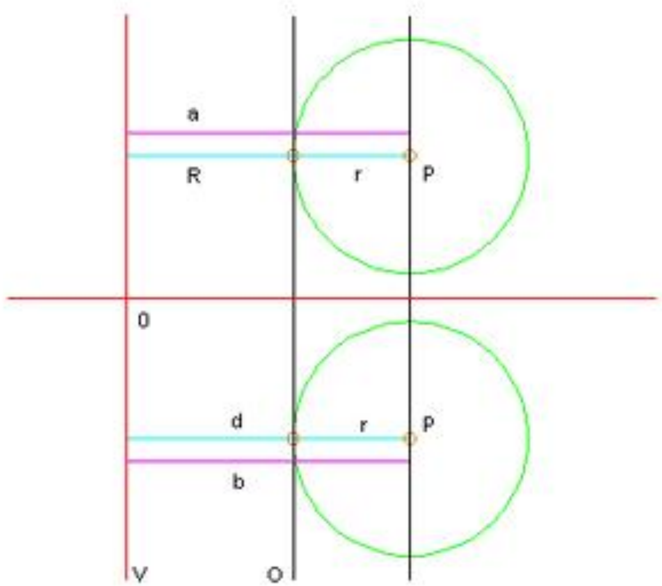
In ogni parabola **R = d** **R** (distanza **F-O**) **d** (distanza **O-V**)

a-R = r **r** (cerchio raccordante)

b-d = r **r** (cerchio raccordante)

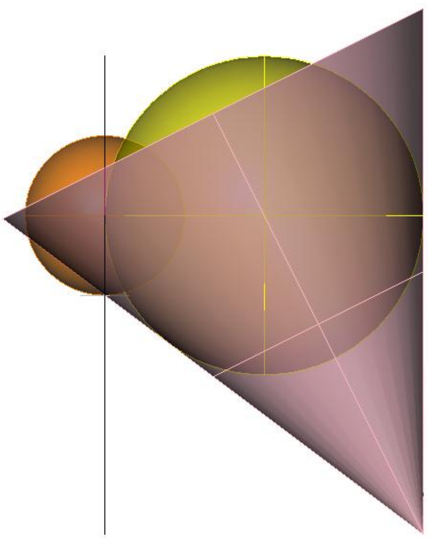
LdE equidistante da cerchio e piano

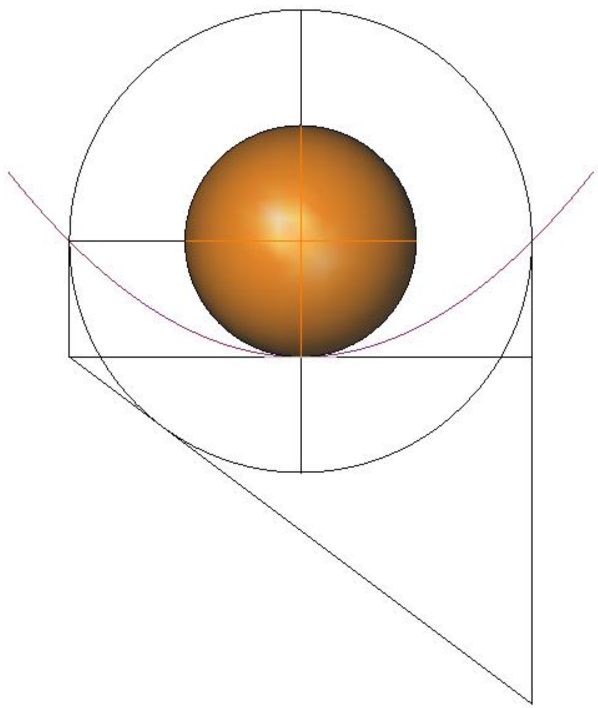


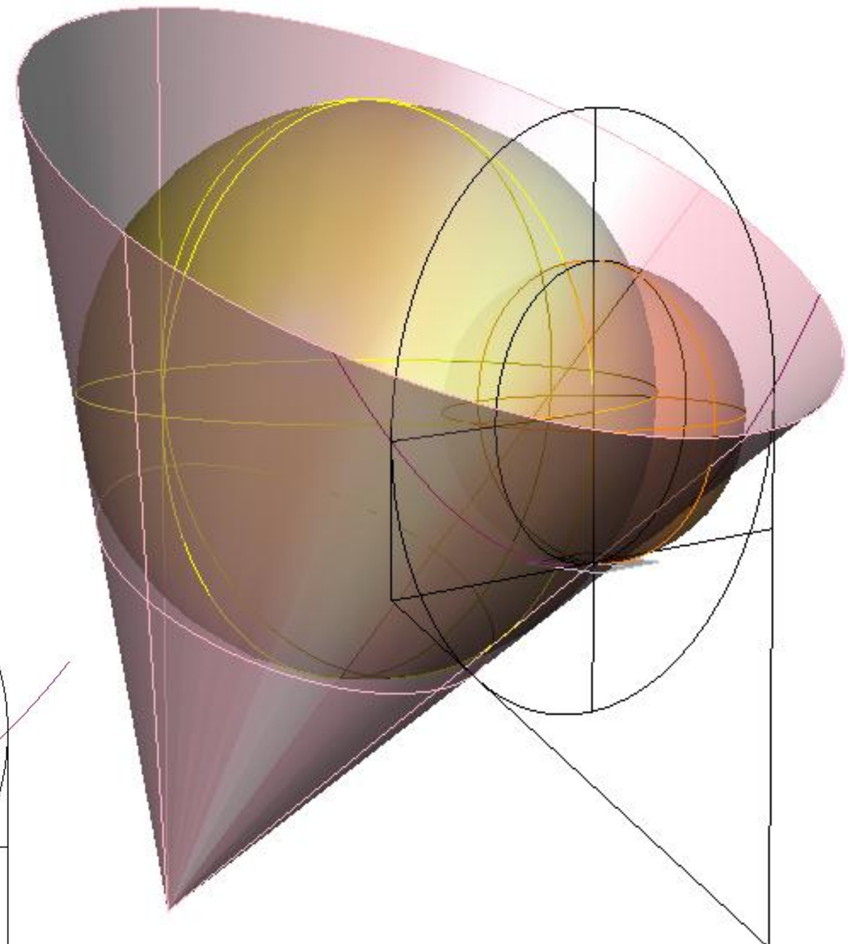


Le **sfere** di **Dandelin**

- per la **parabola 1**

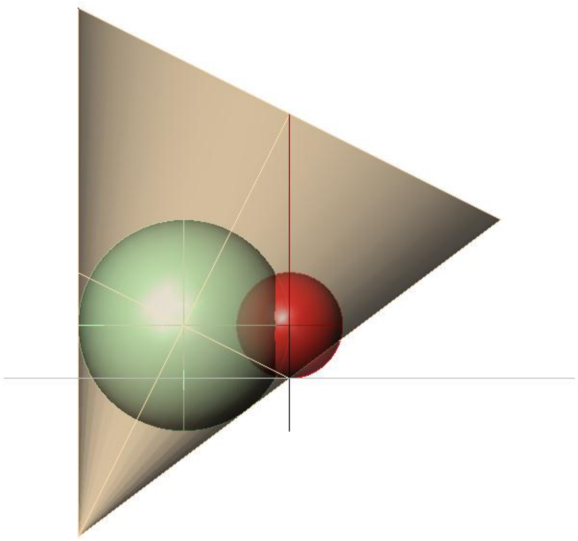


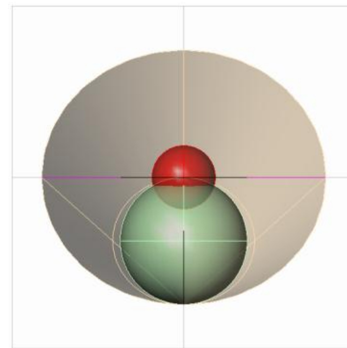


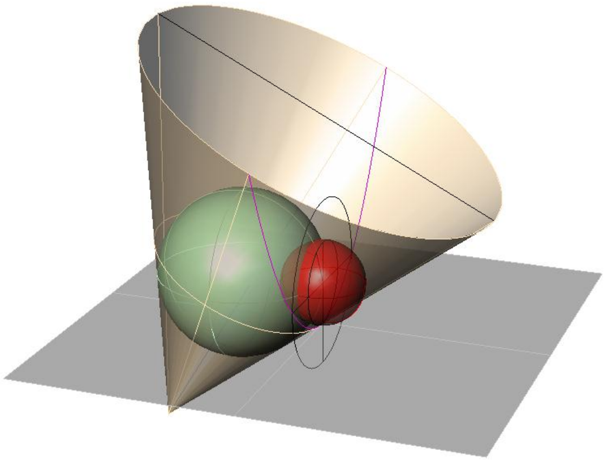


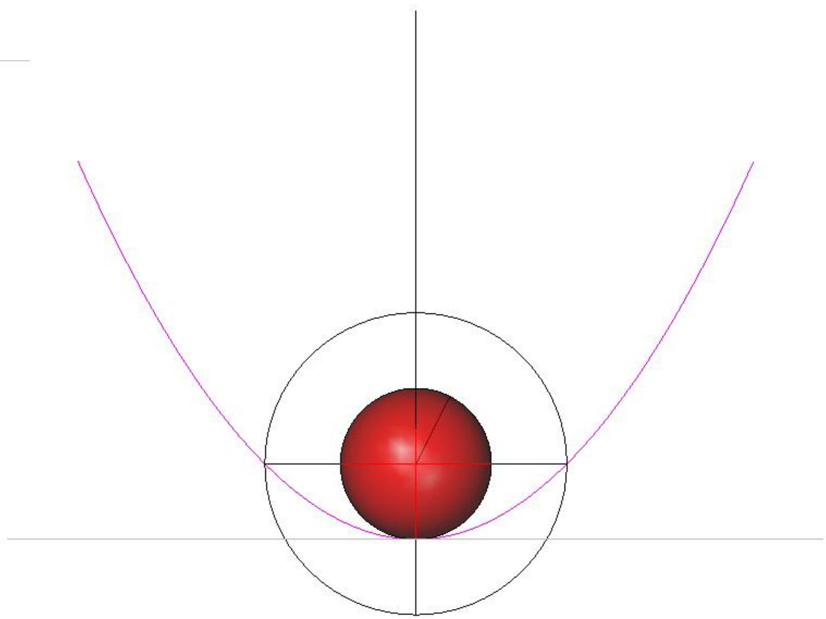
Le **sfere** di **Dandelin**

- per la **parabola 2**

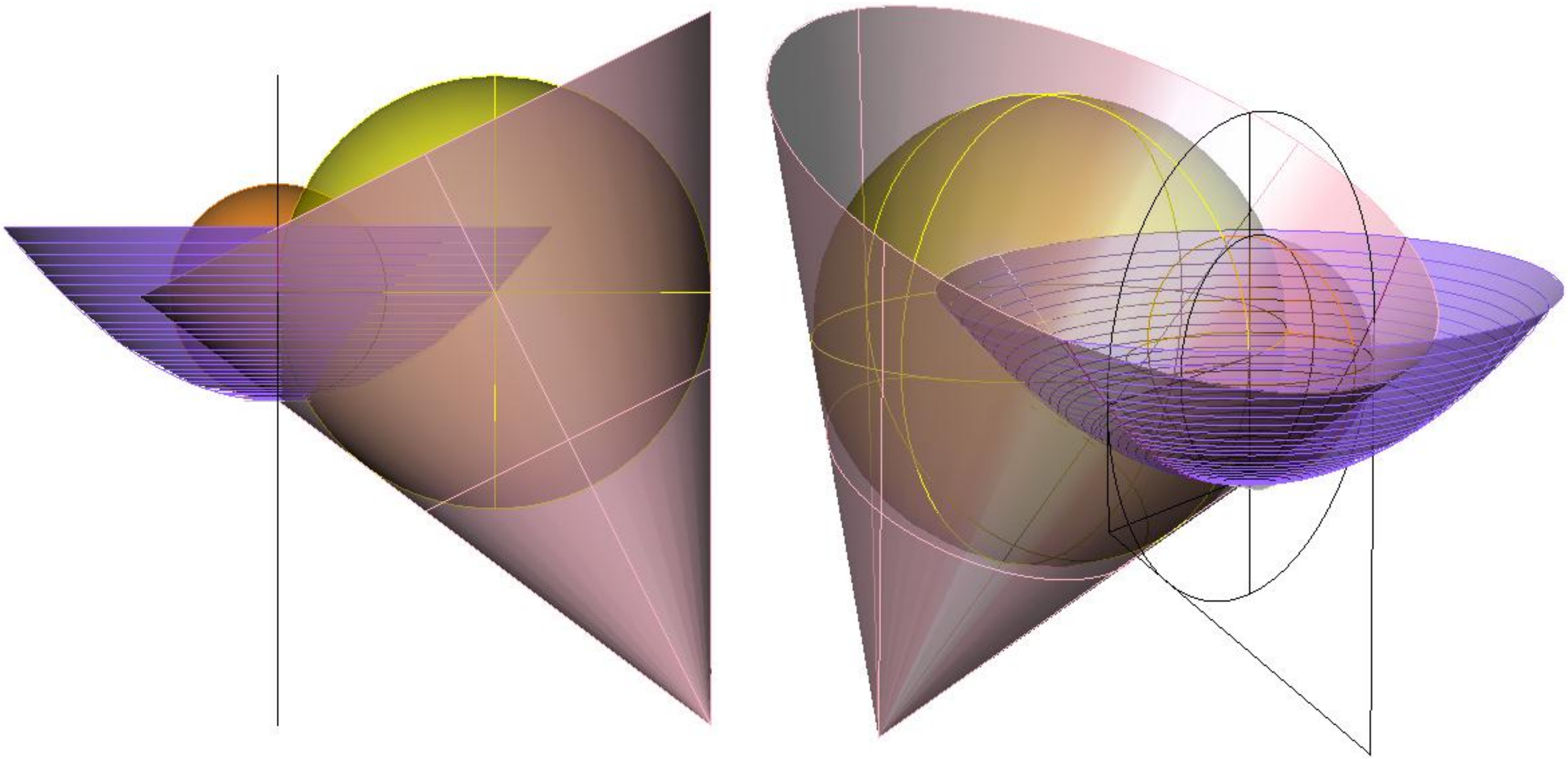




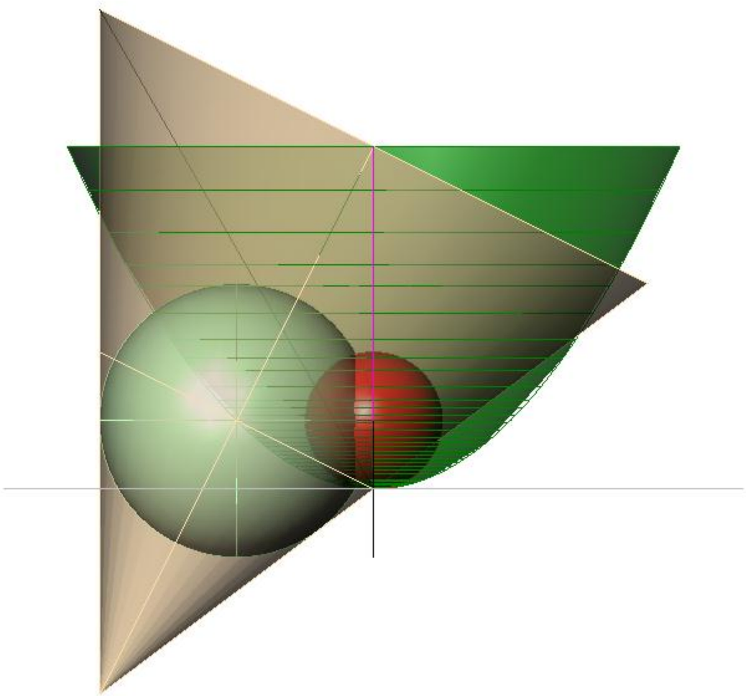


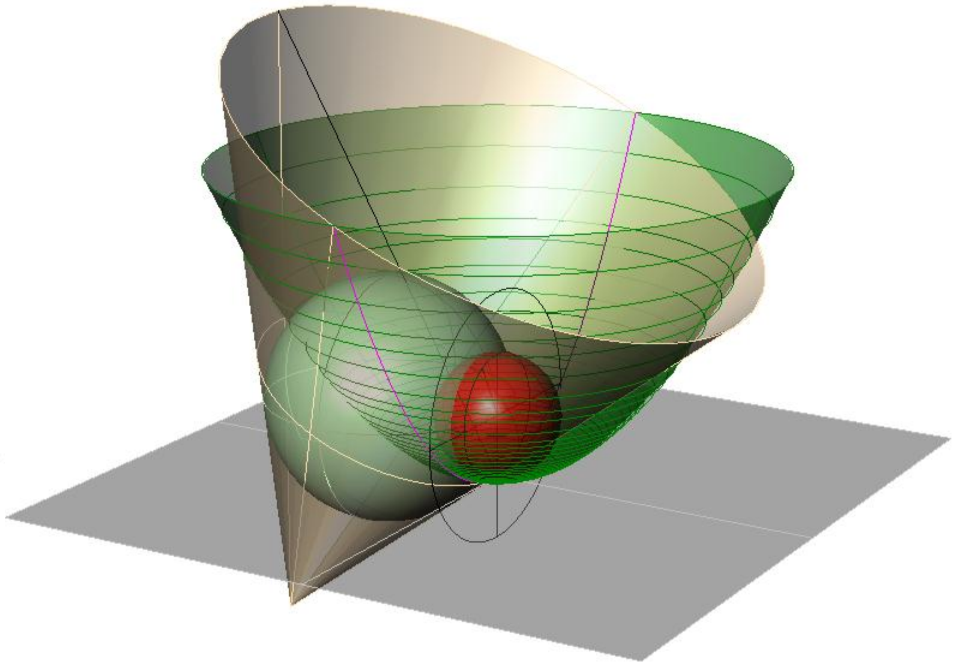


Parabola **1** (**paraboloide 1**)



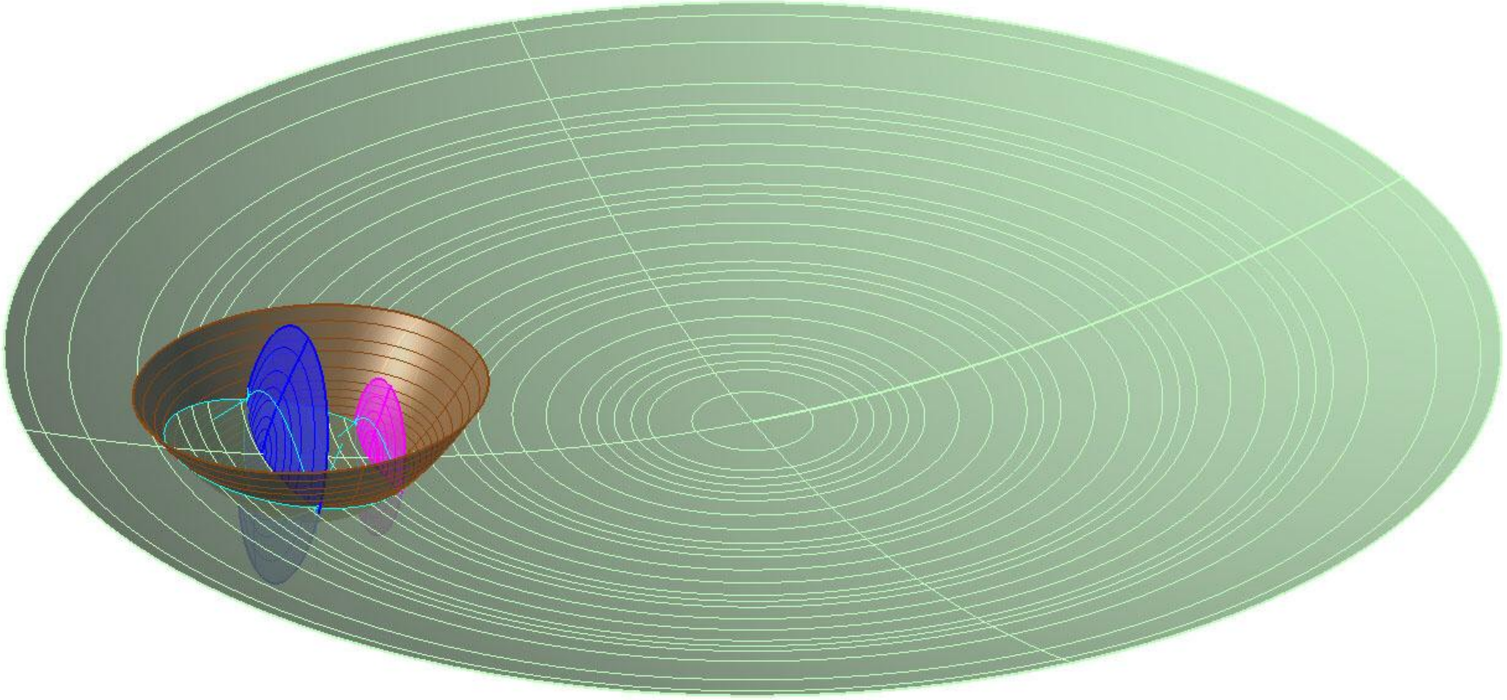
Parabola **2** (**paraboloide 2**)

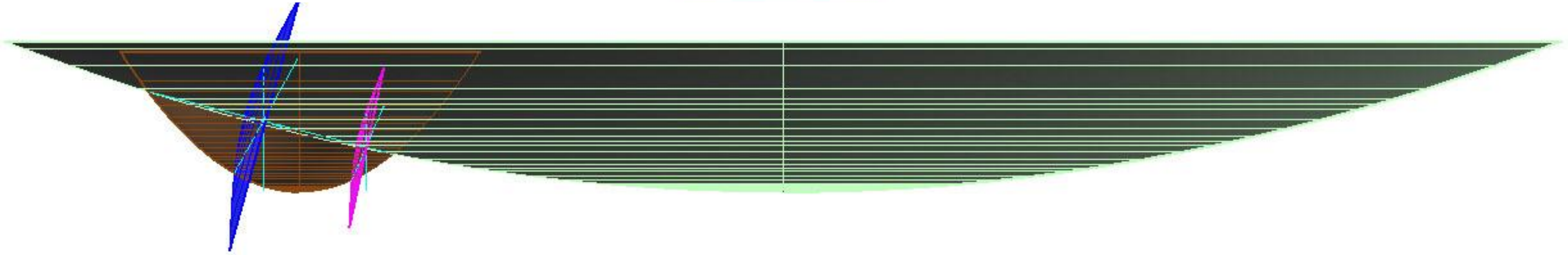




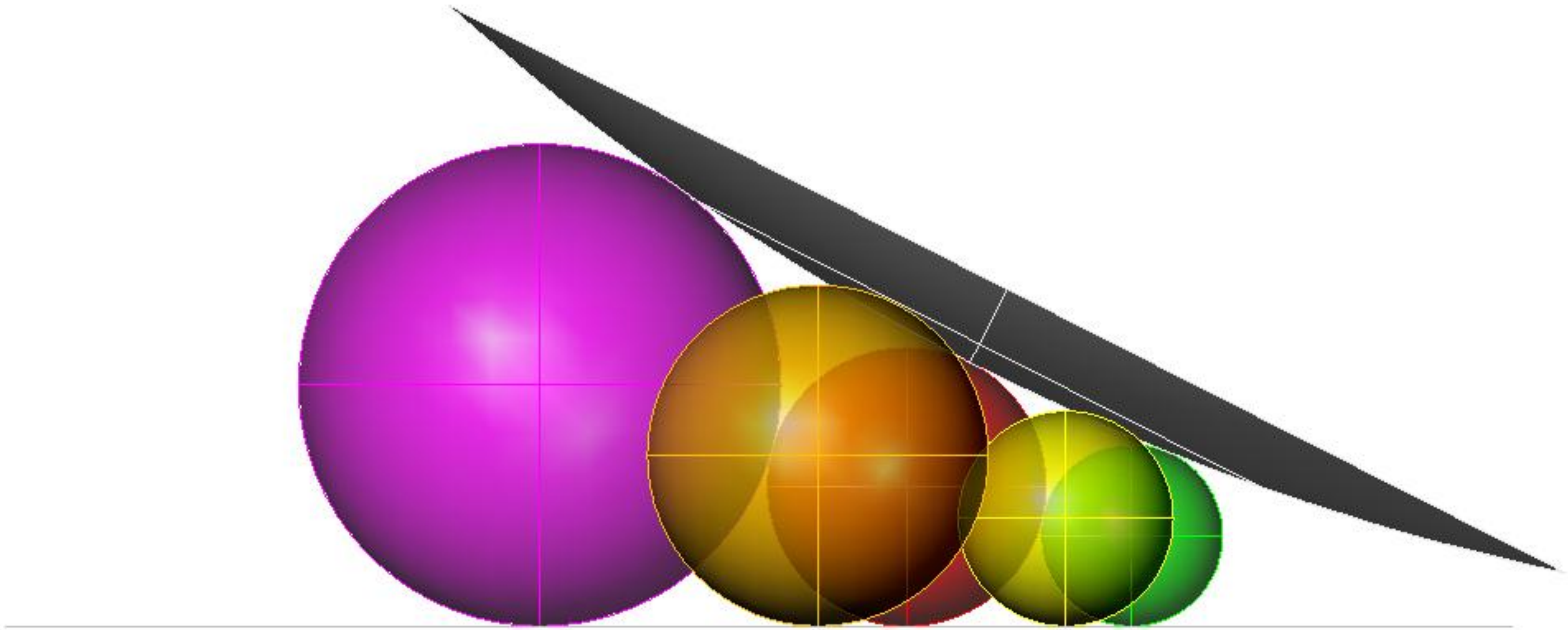
Intersezioni tra oggetti 3D (compresi due iperboloidi)

Determinazione dei raggi delle sfere 6-9





La configurazione può essere costruita



Varie viste